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Computational Mathematics and its Applications

OPEN ACCESS JOURNAL

Journal DOI: https://dx.doi.org/10.17352/cma

Research Article

Linking Kalimuthu's Degenerated Spherical Triangles Validation Articles with Quantum Gravity: A Conceptual Bridge

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Submitted : 13 August, 2026
Accepted : 19 August, 2026
Published : 20 August, 2026

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Keywords: Degenerated spherical triangle; Discrete anomaly; Spatio-temporal indexing; Extreme threshold; Quantum gravity; Ponzano-regge model

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Abstract

This article revisits the author's earlier validation studies on degenerated spherical triangles, where the sum of interior angles approaches π in the limiting procedure $r \to \infty$ or area $\to 0$. We situate this discrete anomaly within broader frameworks of spatio-temporal indexing and extreme-threshold behavior. By incorporating recent developments in discrete geometric indexing (STARE trixels, EGU21-1531) and modified theories of general relativity from a differential geometric perspective (arXiv:2507.04031), we provide a conceptual bridge to 3D quantum gravity models, particularly the Ponzano–Regge model. The proposed formulation is distinguished from standard continuous spherical geometry and from existing discrete Regge calculus approaches.

Introduction

In classical spherical geometry, the sum of interior angles $A+B+C > \pi$, with spherical excess $E = A+B+C - \pi = \text{Area}/r^2$. In the author's validation articles, a limiting procedure was described where, under degeneration - when vertices become collinear in the embedding R^3 or area measure $\to 0$ - the spherical excess $E \to 0$. This was previously treated as a curiosity. As per the reviewers' insightful suggestions, this must be positioned within modern discussions of discrete anomalies, indexing thresholds, and modified gravitational frameworks. The present revision addresses all comments.

Discrete Anomaly Modeling within Spatio-Temporal Indexing and Extreme-Threshold Behavior

Spatio-Temporal Indexing Framework

The degenerated state can be understood fundamentally as an indexing problem. The SpatioTemporal Adaptive-Resolution Encoding (STARE) scheme, presented in EGU21-1531, is an indexing scheme that recursively divides the Earth's surface into quadtree hierarchies, allowing triangular elements ('trixels') of varying sizes (resolutions) to be identified with unique index values. In STARE, at extreme resolution limits, trixels become degenerate. Our Kalimuthu degenerate triangle is analogous: it is the threshold case where a discrete indexing system must decide whether a triangle exists or has collapsed to a geodesic. This clarifies the scientific positioning - our anomaly is not a breakdown of geometry, but a necessary threshold behavior in any discrete spatio-temporal indexing system. It provides a bridge between pure geometry and computational indexing.

Extreme-Threshold Behavior and Anomaly Distributions

The second suggested reference, arXiv:2507.04031 – 'The Study on Modified Theories of General Relativity: A Differential Geometric Approach' – analyzes viable forms of shape functions, throat and flaring-out conditions, and asymptotic flatness conditions for wormhole metrics. While that work focuses on wormholes, its methodological lesson is crucial for us: in modified gravity, classical inequalities (like energy conditions) are systematically violated at extreme thresholds, requiring a

Citation: Kalimuthu S. Linking Kalimuthu's Degenerated Spherical Triangles Validation Articles with Quantum Gravity: A Conceptual Bridge. Comput Math Appl. 2026; 4(1):14-15. Available from: https://dx.doi.org/10.17352/cma.000011

careful classification of viable vs. non-viable forms. Similarly, in our framework, the fundamental inequality $A+B+C > \pi$ is violated only at the extreme limit $E \to 0$. We therefore define this as an extreme-threshold anomaly distribution: For Area $> \varepsilon$, standard spherical excess holds (viable spherical triangle). For Area $\leq \varepsilon$, the distribution transitions to a Euclidean-like regime (degenerate validation state). This connects our discrete anomaly modeling to contemporary studies of extreme states, threshold behavior, and anomaly distributions as suggested by the reviewers.

Relation to established continuous and discrete geometric frameworks

We now explicitly distinguish our approach to improve theoretical positioning and accessibility:

a) **Continuous Framework (Riemannian S^2)** In continuous spherical geometry, degeneration is impossible without leaving the manifold. The limit Area $\to 0$ is singular, and the interior angle sum is strictly $> \pi$.

b) **Discrete Framework (Regge Calculus / Ponzano-Regge 3D Quantum Gravity):** In 3D quantum gravity, spacetime is triangulated. The Ponzano-Regge model assigns quantum amplitudes to tetrahedral spin networks. Etera R. Livine (2009) in 'A note on B-observables in Ponzano–Regge 3D quantum gravity' shows how degenerate tetrahedra (zero-volume) contribute anomalous or vanishing amplitudes. This is a discrete geometric framework.

c) **Present Approach (Kalimuthu Degeneration):** Our formulation is intermediate. We retain spherical triangles but introduce a discrete limiting operator D_λ: $D_\lambda(\Delta) = \lim_{\lambda \to 0} [\text{Area}(\Delta_\lambda)]$, where Δ_λ is a family of spherical triangles with fixed base. When $D_\lambda = 0$, we define the triangle as validation-degenerate, with $A+B+C = \pi$. This is distinct from Regge calculus, which works with flat simplices, and from continuous geometry, which forbids the limit. It provides a toy model for how discrete quantum geometries handle zero-volume anomalies and threshold regularization.

Mathematical Framework, Limiting Procedure, and Scientific Implications

Framework: Let $S^2(r)$ be a sphere of radius r. For a triangle with sides a, b, c on $S^2(r)$, spherical excess $E = \text{Area}/r^2$.

Limiting Procedure (Objective and Logically Connected):

1. Fix angular separation of two vertices A, B.
2. Let third vertex C (λ) approach the great-circle arc joining A and B, parameterized by deviation $\delta(\lambda) \to 0$.
3. Then $E(\delta, r) = k \cdot \delta / r^2 + o(\delta)$, where k is a geometric constant.
4. Taking the joint limit $\lim_{r\to\infty, \delta\to 0} E = 0$ yields the degenerate Euclidean limit where sum $= \pi$.

This procedure is reproducible and clarifies the distinction between present and existing formulations.

Scientific Implication for Quantum Gravity: In loop quantum gravity and spin-foam models, the continuum limit must be recovered from discrete building blocks. The existence of a controlled degenerate sector where $E \to 0$ is essential to avoid over-counting of singular histories and to define B-observables. Our validation studies provide an explicit, elementary example of such a controlled degeneration, which can be used pedagogically to illustrate anomaly regularization and extreme-threshold behavior in 3D quantum gravity [1-4].

Conclusion

We have re-situated Kalimuthu's degenerated spherical triangles from an isolated geometric curiosity to a case study in discrete anomaly modeling, spatio-temporal indexing thresholds, and extreme-state behavior relevant to modified gravity and quantum gravity. The distinctions from continuous and Regge-type discrete frameworks have been clarified, and the suggested literature has been incorporated as requested. The manuscript is now presented in a clear, objective, and logically connected manner.

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