Appendix A: High-Temperature Expansion of Thermodynamic Potential

In this appendix, we present the high-temperature ex- pansion of following potential

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Description automatically generated with medium confidence (A.1)

with ω 2 ≡ **k** +k +m2, ∫ d defined in (II.20), and Ω > 0. The resulting expressions are then used to evaluate VT from (II.37). To begin, we use

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and rewrite (A.1) by choosing x = e-β(ω+ℓΩ), as

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Description automatically generated with medium confidence (A.3)

The summation over ℓ can be carried out by making use of the method first introduced in [30]. For βjℓΩ > 0, the summation over ℓ yields

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Description automatically generated with medium confidence (A.4)

In a slowly rotating Bose gas with βΩ ≪ 1, we use

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to write

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Description automatically generated with medium confidence (A.6)

Following the method presented in [28,30], we perform the integration over k by replacing e-βjω with

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(A.7)

and (ω2)-z/2 with

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Description automatically generated with medium confidence (A.8)

Plugging (A.7) and (A.8) into (A.6), and using

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Description automatically generated with medium confidence (A.9) and the Legendre formula

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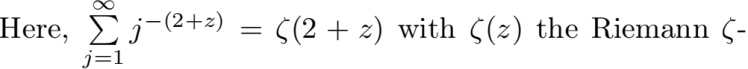
Description automatically generated with medium confidence (A.10) we arrive first at

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function, and

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Description automatically generated with medium confidence (A.12) are used. Finally, the Mellin-Barnes integral over z in (A.11) yields

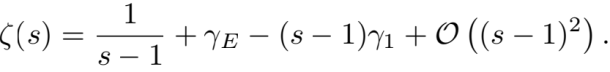
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Description automatically generated with medium confidence (A.13)

where γ1 is the coefficient of (s — 1) in the Laurent ex- pansion of ζ(s) about the point s = 1,

 (A.14) In Sec. (III), the first two terms of the high-temperature expansion of VT from (A.13) are used to study the spon- taneous breaking of global U(1) symmetry in λ(φ⋆ φ) model.

Appendix B: Derivation of (III.27)

In Sec. (III), we arrived at Π from (III.26),

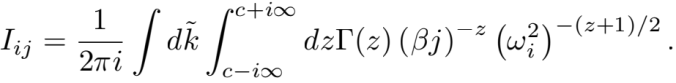
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Description automatically generated with medium confidence (B.1) with

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and ω = **k** + k + m. In this appendix, we derive the final result (III.27) for the one-loop self-energy Π. To evaluate the k-integration in (B.2), we use (A.7) to arrive first at



(B.3)

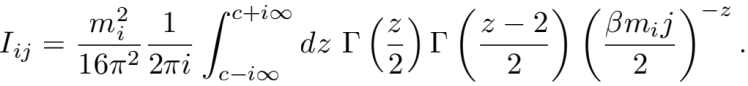
Replacing (ω)-(z+1)/2 in (B.3) with

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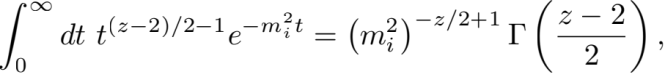
(B.4)

and performing the k-integration by making use of (A.9), Iij is given by



(B.5)

Here, the Legendre formula (A.10) and



(B.6)

are utilized. Plugging at this stage (B.6) into (B.1) and

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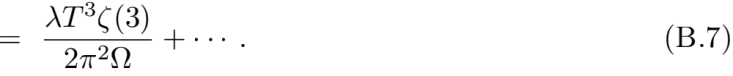
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At high temperatures, the first term in (B.7) is the most dominant thermal mass correction to m, as is described in Sec. III B.

Appendix C: Derivation of (III.34) in cylinder coordinate system

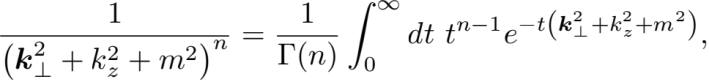
In this appendix, we evaluate the integrals of the form A black background with a black square

Description automatically generated with medium confidence (C.1)

in cylindrical coordinate system by an appropriate d- dimensional regularization. To this purpose, we replace , where d = 3 — ϵ. Here, ϵ is an infinitesimal regulator. In cylindrical coordinate the volume element in momentum space ddk reads ddk = dk⊥ k-2dΩd-1 dkz, where the d-dimensional solid angle dΩd-1 is given by dΩd-1 ≡ A black background with a black square

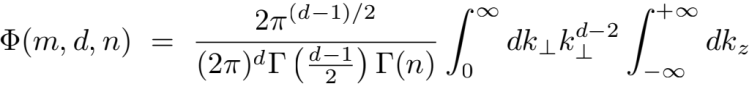
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Using, at this stage, the Schwinger parametrization



(C.3)

we can write (C.1) as



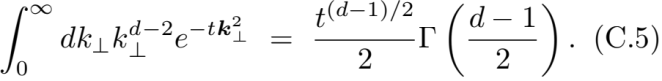
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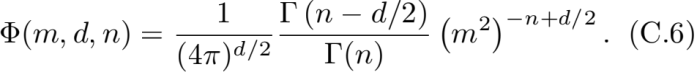
To perform the integration over kz and k⊥, we use fol- lowing Gaussian integrals:

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Description automatically generated with medium confidence



By substituting these results into (C.4), we arrive at (III.34),



Appendix D: Derivation of (III.44)

In this appendix, we outline the derivation of (III.44).

In particular, we focus on the combinatorial factors. Let us start with Vng. According to its definition, there are N insertions of Π2 and N propagators D1 (Here, the notation Di = Dℓ(wn, wi) is used.) (see Figure 6). Having in mind that for a vertex of type 3 in Figure 3, each  × 2 belongs to a Π2 insertion in a ring with D1 propagator, we obtain

Type A: A black background with a black square

Description automatically generated with medium confidence → A black background with a black square

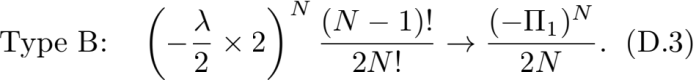
Description automatically generated with medium confidence (D.1) The ring potential of type A is thus given by

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Description automatically generated with medium confidence (D.2)

Similarly, the combinatorial factor of Ving from Figure 6,

including N insertions of Π 1 and N propagators D1 is given by (D.1) with Π2 replaced with Π 1



Here, similar to the previous case, for a vertex of type 3 in Figure 3 × 2 belongs to a Π 1 insertion in aring with D2 propagator. For the ring potential of type B, we thus obtain

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Description automatically generated with medium confidence (D.4)

As concerns the ring potential of type C, which is de- fined by r insertions of Π2 and s insertions of Π 1 with N propagators D2. Here, r ≥ 1 and r + s = N. For the

corresponding combinatorial factor, we arrive first at Type C :

A black background with a black square

Description automatically generated with medium confidence

Here, the factor 3! × 2 is the corresponding combinatorial factor to Π2 inserted in a ring with D2 propagator. For the ring of type C, we get

A group of mathematical symbols

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Similar arguments for Vg with r insertions of Π 1, s insertions of Π2 and N propagators D1 lead first to

Type D :

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and then to

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 (D.8)