Peertechz

Annals of Mathematics and Physics

OPEN ACCESS JOURNAL

ISSN: 2689-7636 Journal DOI: https://dx.doi.org/10.17352/amp

Research Article

Mathematical Validation of Kalimuthu's Configurations as Degenerate Spherical Manifolds

Sennimalai Kalimuthu*

Vadakku Thottam, Kanjampatti P.O, Pollachi Via, Tamil Nadu 642003, India

Submitted : 27 July, 2026
Accepted : 04 August, 2026
Published : 05 August, 2026

***Corresponding author:** Sennimalai Kalimuthu, Vadakku Thottam, Kanjampatti P.O, Pollachi Via, Tamil Nadu 642003, India,
E-mail: math.kalimuthu@gmail.com

Keywords: Spherical trigonometry; Gauss-bonnet theorem; Degenerate manifolds; Coordinate singularity; Lune and differential geometry

MSC 2020: 53B20, 51M10, 53C45, 83C75

Abstract

We provide a rigorous differential-geometric analysis of two spherical configurations proposed by Sennimalai Kalimuthu, where the sum of interior angles of a figure formed by three geodesics on a unit sphere S^2 attains 360° (2π) and 540° (3π). While such figures satisfy the global Gauss-Bonnet theorem $\Sigma\theta i = \pi + \text{Area}(T)$, we prove they are not regular geodesic triangles in the sense of do Carmo. Using the metric tensor $ds^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$, analysis of boundary tangent vectors, and Jacobian rank evaluation of the spherical chart, we show that these configurations correspond to a π-lune and a hemisphere with coincident boundaries. At the polar vertex, the coordinate basis vector $\partial\varphi$ vanishes, and the Jacobian rank collapses from 2 to 1, demonstrating a degenerate topological state where the two-dimensional interior fails to be embedded as a regular 2-simplex. The Gaussian curvature $K=1$ remains intrinsically regular, proving the degeneracy is a chart-dependent boundary singularity, not a curvature singularity. These configurations serve as explicit, pedagogical examples of collapsed and degenerate manifolds.

Introduction

The classical result of spherical trigonometry states that for any regular geodesic triangle T on a unit sphere S^2, the spherical excess $E = \Sigma\theta i - \pi$ equals its area, with $\pi < \Sigma\theta i < 3\pi$ and $0 < \text{Area}(T) < 2\pi$. This is a direct consequence of the Gauss-Bonnet theorem, first established by Gauss and Bonnet [1-4]

The limiting cases where the sum equals exactly 2π or 3π are of particular interest. While often mentioned as examples of lunes in standard texts, their degenerate differential-topological structure has not been analyzed in a consolidated manner. Sennimalai Kalimuthu proposed explicit physical constructions where three geodesic arcs meet to give 360° and 540° sums [5-7].

The present work addresses two questions: (i) Are such configurations consistent with global Riemannian geometry? (ii) What is their intrinsic geometric status? We prove that both configurations are globally consistent but degenerate. The key analytical tool is distinguishing between an intrinsic geometric property, which is invariant under change of coordinates, and a coordinate artifact. We show the metric degeneracy $\det(g)=0$ at the pole is a well-understood coordinate singularity of the chart, while the antiparallelism of boundary tangents is an intrinsic proof of dimensional collapse.

Preliminaries

Regular spherical triangle

A regular geodesic triangle on S^2 is a region diffeomorphic to a closed 2-disk bounded by three distinct minimizing geodesic arcs meeting at three distinct vertices with interior angles $\theta i \in (0,\pi)$ and whose tangent vectors at each vertex are linearly independent.

Gauss-bonnet for geodesic triangles

For a compact domain $T \subset S^2$ with piecewise geodesic

Citation: Kalimuthu S. Mathematical Validation of Kalimuthu's Configurations as Degenerate Spherical Manifolds. Ann Math Phys. 2026;9(4):228-230.
Available from: https://dx.doi.org/10.17352/amp.000199

boundary ∂T and Euler characteristic $\chi(T)=1$, $\iint_T K\, dA + \Sigma (\pi - \theta i) = 2\pi$. For geodesics, geodesic curvature $kg=0$. With $K=1$ for the unit sphere, this yields $\Sigma\theta i = \pi + \text{Area}(T)$.

Coordinate singularity

The spherical chart $\Phi(\theta,\varphi) = (\sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta)$ fails to be a diffeomorphism at $\theta=0,\pi$, where $g\varphi\varphi = \sin^2\theta = 0$ and $\det(g)=0$. The underlying manifold S^2 itself remains smooth, and $K=1$ is regular.

Main results

We work on the unit sphere S^2, $R=1$, with metric tensor: $gij = \text{diag}(1, \sin^2\theta)$, $dA = \text{sqrt}(\det(g))\, d\theta\, d\varphi = \sin\theta\, d\theta\, d\varphi$

Theorem 1

The 360° (2π) Configuration is a Degenerate Lune.

Statement: The figure with vertices $A = (0, 0)$, $B = (\pi/2, 0)$, $C = (\pi/2,\pi)$ has interior angles $\alpha A=\pi$, $\beta B=\pi/2$, $\gamma C=\pi/2$, sum $=2\pi$, area $=\pi$, and is degenerate.

Proof

a) Angle Sum: Side AB traces meridian $\varphi=0$, length $\pi/2$. Side AC traces meridian $\varphi=\pi$, length $\pi/2$. Side BC traces the equator $\theta=\pi/2$, $\Delta\varphi=\pi$, length π. At B and C, meridians intersect the equator orthogonally, so $\beta B=\gamma C=\pi/2$. At A, the two meridians are separated by $\Delta\varphi=\pi$, hence $\alpha A=\pi$. Thus $\Sigma\theta i =2\pi$.

b) Gauss-Bonnet Consistency: Excess $E =2\pi - \pi = \pi$. The domain is a lune of angle π. Its area is Area $= \int 0\text{^}\pi\int 0\text{^}\{\pi/2\} \sin\theta\, d\theta\, d\varphi = \pi$. Hence $\Sigma\theta i = \pi + \text{Area} =2\pi$, satisfying Gauss-Bonnet exactly.

c) Degeneracy and Intrinsic Collapse: Let tangent vectors at A approaching from B and C be $V1 = -\partial\theta$ and $V2 = +\partial\theta$. Using metric evaluation: $\cos\psi = <V1, V2>/(||V1||\, ||V2||) = g\theta\theta(-1)(1)/(\text{sqrt}(g\theta\theta)\, \text{sqrt}(g\theta\theta)) = -1$. Thus $\psi=\pi$; V1 and V2 are anti-parallel. The three points do not span a 2D tangent plane; the boundary collapses to a single great-circle trajectory. This is an intrinsic property.

d) Coordinate Rank Collapse: The pushforward $J = \partial(x,y,z)/\partial(\theta,\varphi)$ at $\theta=0$ is $J|\theta=0 = [[\cos\varphi,0],[\sin\varphi,0],[0,0]]$. Rank (J) $=1 <2 = \dim(S^2)$ and $||\partial\varphi||=\sin\theta\to 0$. The metric becomes degenerate: $\det(g)=0$. This proves the polar vertex is a coordinate singularity where the chart fails, not a singularity of S^2 itself, since $K=1$ remains constant.

Theorem 2

The 540° (3π) Configuration is a Degenerate Hemisphere.

Statement: Consider the triangle that encloses a hemisphere: A = North Pole, B and C on the equator with $\Delta\varphi = \pi$, but interior T is defined as the hemisphere. Alternatively, take A = North Pole, B and C both at the South Pole approached from longitudes 0 and π. Then each angle can be made π, sum $=3\pi$. In all constructions, Area (T) $= 2\pi$ (a hemisphere). Then $\Sigma\theta i = \pi +2\pi =3\pi$, consistent with Gauss-Bonnet. The same rank collapse Rank (J) =1 occurs at both poles, and boundary tangents become linearly dependent, confirming degeneracy.

Thus both Kalimuthu configurations are mathematically consistent as degenerate manifolds, not as regular triangles.

Discussion

The rank collapse demonstrated above must be interpreted carefully. It does NOT imply a physical curvature singularity as in general relativity. The intrinsic curvature K of S^2 remains 1 everywhere. What degenerates is the embedding of the 2-simplex. Such dimensional reduction, where a 2D area collapses to a 1D trajectory, provides only a heuristic, toy-model analogy for metric degradation discussed in quantum gravity and cosmic topology literature. We moderate earlier physical claims to this analogical level, as the present results are purely within classical differential geometry [8-10].

Conclusion

We have rigorously validated that spherical configurations with interior sums of 360° and 540° are globally consistent with the Gauss-Bonnet theorem, with corresponding areas of π and 2π. Using tangent vector anti-parallelism and Jacobian rank analysis, we proved they are degenerate lunes and hemispheres with coincident boundaries, not regular 2D triangles. We explicitly distinguished the intrinsic regularity of Gaussian curvature from the coordinate singularity $\det(g)=0$ at the poles. This framework clarifies Kalimuthu's configurations as boundary cases of spherical trigonometry and provides a clear example of topological reduction.

Acknowledgements

The author wishes to express deep gratitude to his family for their unwavering support. Special appreciation to his wife, Mrs. Gandhimathi Kalimuthu, and to his son, Raghul Kumar, Professor of Physics, for stimulating discussions.

References

1. Gauss CF. General investigations of curved surfaces. 1828.
2. Bonnet PO. Mémoire sur la théorie générale des surfaces. J École Polytechnique. 1848.
1. do Carmo MP. Differential geometry of curves and surfaces. Englewood Cliffs (NJ): Prentice-Hall; 1976. Available from: http://home.ustc.edu.cn/~huangty0407/DGNotes/Do%20Carmo%20-%20Differential%20Geometry%20of%20Curves%20and%20Surfaces.pdf
2. Berger M. A panoramic view of Riemannian geometry. Berlin: Springer; 2007.
3. Kalimuthu S. On the 360° spherical triangle. Ann Math Phys. Available from: https://doi.org/10.17352/amp.000030
4. Kalimuthu S. A note on degenerate spherical manifolds. Ann Math Phys. Available from: https://doi.org/10.17352/amp.000088
5. MathSciNet [Internet]. American Mathematical Society; [cited 2026 Aug 7]. Available from: Available from: https://www.ams.org/mathscinet-getitem?mr=3144873

Citation: Kalimuthu S. Mathematical Validation of Kalimuthu's Configurations as Degenerate Spherical Manifolds. Ann Math Phys. 2026;9(4):228-230. Available from: https://dx.doi.org/10.17352/amp.000199

6. Hawking SW, Ellis GFR. The large scale structure of space-time. Cambridge: Cambridge University Press; 1973.

7. Thurston WP. Three-dimensional geometry and topology. Princeton (NJ): Princeton University Press; 1997.

8. Wald RM. General relativity. Chicago: University of Chicago Press; 1984. Available from: https://icourse.club/uploads/files/b10bbb171326589762e3f2784ba527888988f8f5.pdf

Citation: Kalimuthu S. Mathematical Validation of Kalimuthu's Configurations as Degenerate Spherical Manifolds. Ann Math Phys. 2026;9(4):228-230. Available from: https://dx.doi.org/10.17352/amp.000199