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Annals of Mathematics and Physics

Annals of
Mathematics and Physics
OPEN ACCESS JOURNAL

ISSN: 2689-7636 Journal DOI: https://dx.doi.org/10.17352/amp

Research Article

The Geometry of Collapse: Structural Indeterminacy, Kalimuthu's Extremes, and Boundary Transitions in Degenerate Antipodal Spherical Triangles

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Submitted : 07 July, 2026
Accepted : 27 July, 2026
Published : 28 July, 2026

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Keywords: Antipodal boundaries; Degenerate triangles; Indeterminate identities; Kalimuthu theorem; Spherical excess; Spherical geometry; Straight-angle vertices

MSC: 51M10, 51M20, 51M25 PACS: 02.40.Dr, 02.40.Ky, 04.20.-q, 98.80.Jk

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Abstract

This paper analyzes the geometric breakdown of spherical triangles at antipodal boundary conditions. For the degenerate configuration a=90°, b=90°, c=180°, we show that the spherical Law of Cosines collapses to the indeterminate identity 0=0 and the Law of Sines to the undefined form 0/0. We provide a formal justification by demonstrating that the system possesses a continuous free parameter - the longitude λ of the equatorial vertex - yielding a continuum of valid configurations with identical side lengths. This configuration is presented as a corollary of Kalimuthu's theorem, which permits a closed spherical triangle with angle sum 360° via a straight-angle vertex. Despite angular indeterminacy, Girard's theorem confirms the area remains stable and exactly 1/4 of the sphere (πR^2), bridging classical triangles and multi-angle lunes.

Introduction

The exploration of non-Euclidean spaces challenges flat-space intuitions. While Euclidean triangles have an angle sum of 180°, spherical geometry allows expansion based on curvature. This paper investigates the precise moment at which classical spherical trigonometry experiences structural breakdown when side lengths reach antipodal limits.

When a side reaches 180° (antipodal limit), standard formulas collapse. This is not a failure but an indicator of coordinate indeterminacy. Geometrically, when two vertices settle onto opposite poles, the third vertex can slide along the equator to any longitude without altering boundary arc lengths. The algebra reflects this spatial freedom.

To appreciate this, we incorporate Kalimuthu's spherical geometric theorem. Kalimuthu demonstrated that a closed spherical triangle can have an interior angle sum of exactly 360° by allowing a polar vertex to open into a straight angle (180°).

Materials and methods

A. Formal Statement of Kalimuthu's Theorem (General Form) [NEW - Addresses Reviewer 1]

Kalimuthu's Theorem: On a sphere of radius R, there exists a closed spherical triangle bounded by three great-circle arcs whose interior angle sum is exactly 360°. This is achieved when one vertex attains a straight angle (180°), where two sides become collinear as a single great-circle arc, while the figure remains bounded and closed.

Corollary (Boundary Realization studied in this paper): The configuration with sides a=90°, b=90°, c=180° and opposite angles A=90°, B=90°, C=180° is a degenerate antipodal case of the above theorem. It represents the transition from a closed triangle to a multi-angle lune.

Citation: Kalimuthu S. The Geometry of Collapse: Structural Indeterminacy, Kalimuthu's Extremes, and Boundary Transitions in Degenerate Antipodal Spherical Triangles. Ann Math Phys. 2026;9(4):215-217. Available from: https://dx.doi.org/10.17352/amp.000195

B. Standard Spherical Trigonometric Laws [CORRECTED - Addresses Reviewer 2]

Law of Cosines for Sides: cos c = cos a cos b + sin a sin b cos C

Law of Cosines for Angles: cos C = -cos A cos B + sin A sin B cos c

Law of Sines: sin A / sin a = sin B / sin b = sin C / sin c

Where a,b,c are side arcs (in degrees), and A, B, C are opposite interior angles. Girard's Theorem: Area = E * R^2, where Excess E = A+B+C - 180° (in radians, Area = E_rad * R^2).

Results

Let a=90°, b=90°, c=180° to solve for angle A

cos a = cos b cos c + sin b sin c cos A

cos 90° = cos 90° cos 180° + sin 90° sin 180° cos A

0 = (0)(-1) + (1)(0)cos A

0 = 0

Because sin C = sin 180° = 0, the term containing the unknown cos A is multiplied by zero and drops out. The equation becomes 0=0, independent of A. The Law of Cosines cannot lock down angle A from sides alone.

B. Law of Sines - Indeterminate Form 0/0 [CORRECTED]

Using Law of Sines: sin A / sin a = sin C / sin c

sin A / sin 90° = sin 180° / sin 180°

sin A / 1 = 0 / 0

sin A = 0/0

While the Law of Cosines gave 0=0, the Law of Sines gives 0/0. Both signal the same truth: base angles cannot be determined from side lengths at the antipodal limit.

C. Formal Justification: Existence of Free Parameter and Continuum of Solutions [NEW - Addresses Reviewer 4]

Theorem: The degenerate system possesses a free parameter $\lambda \in [0°, 360°)$ leading to infinite solutions.

Proof: Place Vertex C at the North Pole (90°N), Vertex B at the South Pole (90°S). Place Vertex A on the Equator at longitude λ

Distance b = North Pole to A = 90° for any λ.

Distance c = South Pole to A = 90° for any λ.

Distance a = North Pole to South Pole = 180° for any λ.

Thus side lengths (90°,90°,180°) are invariant under rotation λ. The interior angle at C equals λ (the lune opening angle). Therefore, angle A = 90°, B=90° are not uniquely fixed by sides; A = f(λ). The system has a continuum of solutions parameterized by λ. The algebraic identities 0=0 and 0/0 are exact algebraic images of this geometric degree of freedom, not mere computational errors.

Real-world application: Earth's geographic coordinate system

Mapping: Vertex C = North Pole, Vertex B = South Pole, Vertex A = Equator at Prime Meridian (0°,0°). Side b (North Pole to Equator) = 90° arc.

Side a (North Pole to South Pole) = 180° arc.

Side c (Equator to South Pole) = 90° arc.

Angular Breakdown: At Vertex A, direction to the North Pole is Due North (0°), to the South Pole is Due South (180°). They form a straight line. Because poles are antipodal, Vertex A can slide to any longitude (New York, London, Tokyo) and distances remain 90°,90°,180°. Hence side lengths cannot tell which longitude you are on - algebra gives 0=0.

Area: A wedge bounded by two meridians opening 90° at poles. Since full sphere is 360° longitude, 90° slice = 90/360 = 1/4 of planet. Via Girard: Excess E = 90° = $\pi/2$ rad, Area = E*R^2 = $(\pi/2)R^2$ for single lune; double lune gives πR^2 = 1/4 * $4\pi R^2$ [1-6].

Discussion and conclusion

The collapse of standard formulas into 0=0 and 0/0 is not an error but an elegant built-in signal of indeterminacy. In flat space, side lengths rigidly lock angles; on a curved surface, rigidity shatters at the antipodal limit. Integrating Kalimuthu's theorem as a corollary clarifies this transition: a polar vertex flattening to 180° redefines the traditional corner while remaining structurally closed.

Most importantly, while angular uniqueness vanishes, internal space remains stable and calculable via Girard's theorem. This provides a concrete baseline for understanding spatial transitions at curved-space limits.

Outlook and future work [revised - addresses reviewer 3]

The phenomenon where individual predictability vanishes but the overall system maintains stability may be of interest as an analogy in other fields: e.g., coordinate singularities in cosmology, high-pressure molecular bonds, or resilient engineering structures that shed rigid constraints. Future work may explore these analogies formally.

Declarations

Funding: No external funding. Independent scholarly endeavor.

Conflict of Interest: None declared.

Acknowledgments: My family, my wife, Mrs. Gandhimathi Kalimuthu, and my son, Prof. Raghul Kumar, for encouragement.

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The author is very grateful for the most valuable review comments of the anonymous referees. Indeed, these evaluations have added merit to this article.

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