



Short Communication

# Primacohedron: Prime-Indexed Logarithmic Cosmological Signatures from Emergent Arithmetic Spacetime

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We present a concise observational consequence of the Primacohedron framework, in which spacetime emerges from prime-indexed adelic spectral coherence. The central prediction is a hierarchy of logarithmic modulations in cosmological observables whose characteristic frequencies are tied to the dominant coherent prime sectors. In this revised version, the modulation formula is motivated by an explicit prime-sector transfer kernel rather than stated only phenomenologically. Unlike generic resonant inflationary features, the predicted frequencies are discretized by arithmetic structure and recur coherently across scalar perturbations, tensor backgrounds, and entropy-linked non-Gaussian observables. The framework also predicts mild late-time deviation from exact vacuum dark energy and a running spectral dimension from approximately two in the ultraviolet toward four in the infrared. A concrete restricted-prior statistical pipeline is specified, including prime selection, model comparison, look-elsewhere control, and cross-sector recurrence tests. The decisive observational test is correlated prime-log recurrence across sectors.

## 1. Emergent arithmetic spacetime

Standard cosmology successfully describes large-scale observations using the  $\Lambda$ CDM framework supplemented by inflationary dynamics [1-4]. However, the microscopic origin of inflation, dark energy, and spacetime dimensionality remains unresolved.

The Primacohedron framework reverses the conventional order of explanation: spacetime is not fundamental but emerges from synchronized prime-indexed spectral sectors [5]. Each

prime  $p$  contributes a local non-Archimedean resonance sector, while macroscopic geometry appears through adelic coherence [6-8]. The working assumption is not that ordinary matter directly “knows” the arithmetic identity of each prime. Rather, the fundamental spectral object is taken to be an adelic operator

$$\mathcal{L}_{ad} = \mathcal{L}_{\infty} \oplus \bigoplus_{p \in \mathcal{P}} \mathcal{L}_p,$$

Where  $\mathcal{L}_{\infty}$  describes the Archimedean, continuum sector, and  $\mathcal{L}_p$  describes a local  $P$ -adic resonance sector. Coherence means that low-energy perturbations couple to the diagonal, adelically matched sector. Prime labels then enter the effective perturbation theory through the eigenphase increments of local dilation modes, for which the natural logarithmic generator has frequencies proportional to  $\ln p$ .

In the effective cosmological limit, the expansion dynamics take the schematic form

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_{arith}}{3} \rho_{info} - \frac{k}{a^2} + \frac{\Lambda_{arith}}{3},$$

where  $\rho_{info}$  denotes an effective information-energy density generated by spectral coherence gradients.

This expression is obtained by coarse-graining the coherence functional

$$\mathcal{I}[g, \Psi] = \sum_p w_p \|\nabla_{\log a} \Psi_p\|^2 + V_{coh}(\Psi)$$

into an isotropic effective stress tensor. The energy density contribution is



$$\rho_{info} = a^{-3} \sum_p w_p \|\nabla_{\log a} \Psi_p\|^2 + V_{coh},$$

so the Friedmann-like equation should be read as an effective semiclassical limit, not as a derivation from a conventional scalar-field Lagrangian. When  $p_{info}$  varies slowly over a Hubble time,

$$a(\tau) \sim a_0 \exp\left[\left(\frac{8\pi G_{arith} \rho_{info}}{3}\right)^{1/2} \tau\right].$$

Inflation therefore arises from entropy-curvature amplification rather than from a fundamental inflaton potential. The precise claim is modest: the mechanism supplies an effective quasi-de Sitter phase if the coherence energy is approximately constant and positive. The paper does not require every inflationary observable to be determined by this schematic sector alone.

## 2. Prime-log modulation

The main observational prediction concerns the primordial scalar spectrum. Starting from a smooth baseline spectrum,

$$P_{\mathcal{R}}^{(0)}(k) = A_s \left(\frac{k}{k_0}\right)^{n_s - 1 + \frac{1}{2} \alpha_s \ln(k/k_0)},$$

the Primacohedron correction introduces a discrete logarithmic modulation,

$$P_{\mathcal{R}}(k) = P_{\mathcal{R}}^{(0)}(k) \left[ 1 + \sum_{p \in \mathcal{P}^*} \lambda_p \cos\{(\ln p) \ln(k/k_0) + \phi_p\} \right].$$

A minimal route to this expression is as follows. Let  $x = \ln(k/k_0)$  and suppose the curvature transfer function contains a small adelic correction

$$T(x) = T_0(x) \left[ 1 + \epsilon \sum_{p \in \mathcal{P}^*} c_p e^{i(\ln p)x} + c_p^* e^{-i(\ln p)x} \right].$$

Because  $P_{\mathcal{R}} \propto |T|^2$ , keeng terms to first order in  $\epsilon$  gives

$$P_{\mathcal{R}}(k) = P_{\mathcal{R}}^{(0)}(k) \left[ 1 + \sum_{p \in \mathcal{P}^*} \lambda_p \cos((\ln p)x + \phi_p) \right],$$

with  $\lambda_p = 2\epsilon |c_p|$  and  $\phi_p = \arg c_p$ . The formula is therefore a first-order perturbative template generated by prime-sector dilation phases.

The key point is that the oscillation frequencies are not arbitrary free parameters. They are constrained by prime logarithms  $\ln p$ .

The finite set  $\mathcal{P}^*$  is not the set of all primes. It denotes the subset whose coherence weights  $w_p |c_p|$  survive smoothing over the observational window. A practical prior is to begin with the first few primes, for example  $p = 2, 3, 5, 7, 11, 13$ , and then penalize additional primes by an information criterion. This avoids converting the model into an unrestricted multi-frequency Fourier fit.

The expected modulation amplitude is small, typically  $\lambda_p \sim 10^{-3} - 10^{-2}$ , potentially detectable in combined CMB, large-scale structure, and future 21-cm observations.

Smooth Spectrum  $\rightarrow$  Prime-Log Ripple Hierarchy

Scalar and tensor sectors exhibit compatible logarithmic spacing governed by  $\ln p$

## 3. Tensor and dark-energy sector

The same prime-indexed coherence sectors affect tensor perturbations. A stochastic gravitational-wave background may acquire logarithmic fine structure,

$$\Omega_{GW}(f) = \Omega_0 \left(\frac{f}{f_0}\right)^{n_T} \left[ 1 + \delta_{arith} \cos((\ln p^*) \ln(f/f_0) + \phi_T) \right].$$

The revised expression uses the same log-frequency convention as the scalar template. The tensor phase  $\phi_T$  need not equal the scalar phase, because scalar and tensor transfer functions project the same prime-sector kernel through different late-time propagation effects. The recurrence test therefore concerns the frequency set  $\{\ln p\}$  and coherence weights, not exact equality of phases.

The important prediction is not merely primordial tensors, but cross-sector recurrence: scalar and tensor spectra should exhibit compatible logarithmic spacing patterns [9,10].

Late-time acceleration similarly reflects incomplete arithmetic saturation. The effective dark-energy equation of state takes the schematic form

$$w_{DE} = -1 + \frac{\langle F \rangle}{3H^2 M_p^2},$$

yielding a stable small deviation from exact vacuum behavior, for example  $w_{DE} \approx -0.98$ .

Here  $F$  is defined as the residual coherence-pressure functional

$$F \equiv \rho_{coh} + p_{coh},$$

where  $\rho_{coh}$  and  $p_{coh}$  are the effective energy density and pressure after the prime-sector coherence field is coarse-grained. For a perfect-fluid component,  $w + 1 = (\rho + p) / \rho$ . Identifying the dominant late-time denominator with  $\rho_{DE} \simeq 3H^2 M_p^2$  gives the displayed estimate. Thus  $F = 0$  corresponds to exact vacuum behavior, whereas incomplete arithmetic saturation leaves a small positive or negative residual. The numerical value  $w_{DE} \approx -0.98$  is not asserted as a fixed prediction; it is an illustrative target-scale deviation that current and future dark-energy surveys could constrain.

## 4. Running spectral dimension

The framework also predicts scale-dependent effective dimensionality. Using the heat-kernel definition,

$$d_s(\ell) = -2 \frac{d \ln \text{Tr} e^{-\ell^2 L}}{d \ln \ell^2},$$

the effective spectral dimension evolves approximately as

$$d_s(\ell) \simeq 2 - \alpha \ln\left(\frac{\ell}{\ell_p}\right),$$

approaching  $d_s \simeq 2$  in the ultraviolet and restoring toward four dimensions in the infrared.

The sign in this compact expression depends on the convention for the reference scale. To remove ambiguity,



define  $u = \ln(\ell / \ell_p)$  and write the interpolating form

$$d_s(\ell) = 2 + \beta u + O(u^2), \beta > 0,$$

for the near-ultraviolet regime before saturation. More generally, let the heat trace of the effective adelic Laplacian have the small-scale form

$$K(\ell) = \text{Tr} e^{-\ell^2 L_{\text{eff}}} \simeq C \ell^{-2} \exp[-\gamma(\ln(\ell / \ell_p))^2],$$

where  $L_{\text{eff}}$  is the coarse-grained Laplace operator induced by the adelic spectral measure. Then

$$d_s(\ell) = -2 \frac{d \ln K}{d \ln \ell^2} = 2 + 2\gamma \ln(\ell / \ell_p),$$

so the spectral dimension starts near two and increases logarithmically until the infrared completion drives it toward four. The role of the prime spectrum is encoded in the departure of the spectral counting measure from the purely continuum Weyl law.

Although dimensional running appears in several quantum-gravity approaches, the Primacohedron uniquely ties the transition to prime-density saturation and arithmetic spectral structure.

### 5. Comparison with existing feature models

Conventional inflationary cosmology can generate oscillatory features through engineered inflaton potentials, transient sound-speed variations, resonant axion-like couplings, turns in multi-field field space, or standard-clock dynamics [11- 14]. The earlier version understated this point. Existing feature models may contain multiple frequencies, damped oscillations, phase correlations, or correlated signatures in the power spectrum and bispectrum. They are therefore serious comparators rather than straw-man alternatives.

The distinction proposed here is narrower and more testable. In resonant or monodromy-type models, the frequency set is determined by the period and shape of a potential or by the mass scale of a clock field. In the Primacohedron template, the allowed frequencies are restricted before fitting to the arithmetic set  $\omega_p = \ln p$ . The advantage is not greater flexibility, but less freedom: a detection at an arbitrary log-frequency would not support the model unless it aligns with the prime-log restricted family and recurs across sectors.

The Primacohedron differs in two respects. First, the logarithmic frequencies are discretized by an arithmetic structure and tied to  $\ln p$ . Second, scalar, tensor, and non-Gaussian sectors are linked by a common coherence mechanism rather than being independently adjustable. Accordingly, the paper should be read as proposing a restrictive observational template, not as claiming that other feature models cannot generate complicated spectra (Table 1).

### 6. Falsifiability and statistical pipeline

The framework is strongly falsifiable. If future high-precision cosmological observations show no evidence of

Table 1: Technical comparison with common primordial-feature mechanisms.

Model class	Frequency origin	Key discriminator
Axion monodromy / resonant features	Potential periodicity or resonance scale	May allow several frequencies; not restricted to $\ln p$
Sharp or transient features	Time-localized background event	Usually produces a phase/damping tied to event time
Standard-clock signals	Massive field oscillation scale	Frequency tied to mass and expansion history
Primacohedron template	Prime-sector dilation phases $\ln p$	Restricted arithmetic frequencies and cross-sector recurrence

correlated logarithmic spacing across scalar and tensor sectors, the cosmological realization of the Primacohedron is severely constrained.

A concrete analysis pipeline is as follows. First, fit a baseline  $\Lambda$ CDM or extended smooth-spectrum model to the data and construct residuals in  $x = \ln(k / k_0)$  or  $x = \ln(f / f_0)$ . Second, choose a pre-registered prime set  $\mathcal{P}_N = \{2, 3, 5, \dots, p_N\}$ , with  $N$  fixed before unblinding or selected by a penalized criterion such as AIC, BIC, or Bayesian evidence. Third, fit the restricted linear template.

$$r(x) = \sum_{p \in \mathcal{P}_N} [a_p \cos((\ln p)x) + b_p \sin((\ln p)x)]$$

using the full covariance matrix of the data. Fourth, quantify improvement by  $\Delta\chi^2$ , Bayes factor, or posterior predictive loss relative to both the smooth model and an unrestricted multi-frequency feature model. Fifth, control the look-elsewhere effect by Monte Carlo simulations generated under the baseline cosmology and analyzed with the identical prime-restricted search.

Cross-sector recurrence can then be quantified by a coherence statistic.

$$C_{ST} = \frac{\sum_{p \in \mathcal{P}_N} \hat{A}_{p,S} \hat{A}_{p,T} / (\sigma_{p,S} \sigma_{p,T})}{\left[ \sum_p (\hat{A}_{p,S} / \sigma_{p,S})^2 \sum_p (\hat{A}_{p,T} / \sigma_{p,T})^2 \right]^{1/2}},$$

where  $\hat{A}_{p,S}$  and  $\hat{A}_{p,T}$  are fitted scalar and tensor amplitudes. The same statistic may be extended to a bispectrum or squeezed-limit non-Gaussian template. A null result means no statistically significant prime-restricted improvement after covariance, model-complexity penalty, and trials correction.

A practical observational strategy, therefore, consists of:

1. searching for logarithmic residuals using the restricted prior  $\omega_p = \ln p$ ;
2. pre-registering or penalizing the number of included primes;
3. testing whether scalar-sector candidates predict corresponding tensor-sector spacing;
4. checking whether the same coherence parameter improves squeezed-limit non-Gaussian fits;
5. Rejecting the cosmological realization if the prime-restricted template fails against smooth and generic-feature alternatives.



The distinctive feature is therefore not any single anomaly, but coherent arithmetic recurrence across observables.

## Conclusion

The Primacohedron framework predicts that cosmological spectra inherit logarithmic residues from prime-indexed adelic coherence sectors. The decisive empirical signature is correlated prime-log recurrence across scalar perturbations, tensor backgrounds, and entropy-linked observables.

The revision clarifies that the manuscript advances an effective, falsifiable template rather than a completed microscopic theory. The added derivations identify the assumptions behind the prime-log modulation, define the dark-energy residual, derive a heat-kernel route to spectral-dimension running, and specify a statistical pipeline capable of rejecting the model.

This prediction is experimentally vulnerable and therefore scientifically useful. The absence of such correlated arithmetic spacing in future precision cosmological data would strongly constrain this emergent arithmetic-spacetime scenario.

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