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Review Article

Einstein's Equivalence Principle

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Abstract

Based on Newton's laws and Mach's principle, formulas were derived for the quantitative interdependence of inertial and gravitational mass on Earth, other planets, and in space, explaining Einstein's equivalence principle. It was shown that gravitational mass depends on the velocity of a body in the same way as inertial mass.

Introduction

The equivalence principle played an important role in the development of the theory of gravitation. Newton considered this principle so fundamental in mechanics that he devoted the first paragraphs of his Mathematical Principles of Natural Philosophy [1] to a detailed discussion of it. There, he also cited the results of experiments with a pendulum, which he conducted to test this principle. According to Newton, the equivalence principle requires that the mass of any body, namely that property of the body (inertia) that determines its response to an applied force, be equal to its weight – the property that determines its response to gravity [2]. In most theories expressing their attitude to this principle, there were two traditions [3]. The first consisted of interpreting the equality of gravitational and inertial masses $m_G = m_{in}$ as a fundamental empirical or phenomenological position, which, like the Law of Universal Gravitation as a whole, must be explained based on certain etheric-mechanistic concepts, and after the establishment of the electromagnetic program, based on electromagnetism. The second, significantly less developed tradition dates back to K. Neumann [4] and Mach. It is associated not with attempts to explain this equality, but with its use for restructuring the theory of inertia in classical mechanics. Einstein adopted this second tradition. His merit consisted in abandoning attempts at a mechanical, electromagnetic, or any other explanation of the fundamental equality, accepting it as the starting point for constructing a theory of gravitation. Thus, the equality of these masses is essentially postulated [3]. The goal of this work is to find the quantitative dependence of inertial mass on gravitational mass (gravimass) and to reveal the dependence of the gravimass of a body on its velocity.

Inertial and gravitational mass

Inertial mass m_{in} is determined by Newton's second law:

$$F = m_{in}a \tag{1}$$

Where F – is a force external to the object (impact, pressure, gravitational, magnetic, etc.), m_{in} – is a coefficient with the dimension of mass and is called *the inertial mass*, and a – is the acceleration acquired by the object due to the force acting on it F. As can be seen from equality (1), for a positive inertial mass, if the acceleration vector coincides with the direction of the force vector F, then the acceleration has a positive sign, and if the acceleration of the body is directed against the force acting on it, then the acceleration has a negative sign. For a negative object mass, on the contrary, if the acceleration vector coincides with the direction of the force vector acting on the object, then the acceleration is negative, and if the acceleration vector of the object is directed against the force acting on it, then the acceleration of the object has a positive value. But the

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acting real force F – must also have an opposing force, which compensates for the effect of the force F, which is called *the force of inertia* F_{in}. According to D'Alembert's Principle (1717–1783), the force of inertia determines the property of bodies to maintain the state in which they are [5,6]. Over time, the formulation of the principle has changed. The most relevant version was proposed by Appel [7]: "A vector numerically equal to the product of mass and acceleration and directed opposite to acceleration is called the force of inertia, although this will in no way be a force applied to a point." Then equation (1) for the force of inertia, compensating for the effect of an external force F, will look like:

$$F_{in} = -m_{in}a \ , \tag{2}$$

F_{in} – inertial force such that:

$$| F_{in} | = | F | \tag{3}$$

$$F_{in} = -F \tag{4}$$

Unlike Newton's second law (1), the law defined by *d'Alembert's* principle, in accordance with formula (2), can be conventionally called *d'Alembert's law*. Thus, Newton's law determines the dependence of a body's acceleration on the action of an external force, while *d'Alembert's* law determines the similar dependence of the same body's acceleration on the inertial force arising from the action of an external force on the body.

Gravitational mass is determined by Newton's Law of Universal Gravitation

$$F_G = \gamma \frac{m_{pas} \cdot m_{ac}}{r^2} \ , \tag{5}$$

F_G – the force of attraction between a body A and an active gravitational mass m_{ac} (determines the force of attraction of a body to other bodies) and body B with passive gravitational mass m_{pas} (determines the force of attraction of this body to another body); r – the distance between the bodies; γ – the gravitational constant [2,8]. If the masses of the gravitationally interacting bodies are not equal to each other, then it is obvious that the larger one will be more active and will attract the body with the smaller mass more strongly. If the masses of the bodies are equal to each other, then they will attract or repel each other with the same force. If the interacting bodies of the same mass are made of materials of different chemical composition, then since the causes of gravitational forces can have different dependence on the fields that form them, different in different materials, so that m_{pas} could differ from m_{ac}. However, experiments, with an error of 10^{-15}, have shown [9,10] that:

$$m_{pas} = m_{ac} \tag{6}$$

Formula (5), taking into account (6), at least for bodies consisting of the same material, can be written in the form:

$$F_G = \gamma \frac{m_G^A \cdot m_G^B}{r^2} \ , \tag{7}$$

Where m_g^A and are m_g^B – the gravitational masses of the bodies A and B respectively.

The principle of equivalence of inertial and gravitational masses

Mach's principle [11,12] consists of relating the inertial motion of a body not to Absolute Space (AS), but to a frame of reference associated with distant stars, more precisely, to the center of mass of the Universe from the moving body to the body with mass m_i:

$$\frac{d^2}{dt^2}\left[\frac{\sum m_i \boldsymbol{r}_i}{\sum m_i}\right] = 0 \tag{8}$$

Naturally, this automatically required an infinite propagation speed of interactions. Mach's principle itself is obvious. In a deterministic universe, everything must influence everything else. But Newton's principle, which relates motion to the event horizon, is also impeccable, since where r_i – distances are concerned, the existence of the event horizon, if it does not affect the material world, is equivalent to its non-existence. Therefore, it is natural that, formally, both these principles must be taken into account simultaneously, i.e., to determine the force of influence, their additive sum must be taken. This is especially important if the event horizon turns out to be a substantial medium, possessing resistance to the motion of bodies. Thus, Newtonian mechanics, ignoring the influence of distant stars, is, to a certain extent, approximate. The principle of action at a distance, as a requirement for an infinite propagation speed of interactions, does not contradict the principle of action at a short distance, according to which the speed of interaction does not exceed the speed of light. The effects of objects located beyond the event horizon will be delayed, but they will occur. As a result, the distance between the body and the center of mass of the Universe will increase over time, the effective mass of the Universe, consisting of objects that can influence the body, will also increase, and overall, a slight corresponding drift of the final result will be observed over time.

If we assume that C a force acts on a body F with gravitational mass m_G in Absolute Space, in which there are no bodies, then this force will cause the acceleration of this body

$$a_1 = \frac{F}{m_G} \tag{9}$$

The influence of the Universe on the body C in accordance with (7), is determined by the force:

$$F_0 = -\gamma \frac{M \cdot m_G}{R^2} = m_G a_2 \ , \tag{10}$$

where is M – the mass of the Universe, R – the distance from the body C to the center of mass of the Universe, from which we obtain:

$$a_2 = -\gamma \frac{M}{R^2} \tag{11}$$

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$$a = a_1 + a_2 = \frac{F}{m_G} - \frac{\gamma \frac{M \cdot m_G}{R^2}}{m_G} \quad (12)$$

$$m_G a = F - \gamma \frac{M \cdot m_G}{R^2} \quad (13)$$

Since Newton's second law is written in formula (1), then

$$a = \frac{F}{m_{in}} \quad (14)$$

Substituting (14) into (13), we obtain:

$$\frac{m_G}{m_{in}} F = F - \gamma \frac{M \cdot m_G}{R^2} \quad (15)$$

$$\frac{m_G}{m_{in}} = 1 - \gamma \frac{M \cdot m_G}{FR^2} \quad (16)$$

$$m_{in} = \frac{m_G FR^2}{FR^2 - \gamma M \cdot m_G} \quad (17)$$

$$m_G = \frac{m_{in} FR^2}{FR^2 + \gamma M \cdot m_{in}} \quad (18)$$

Formulas (17,18) represent the dependence of inertial mass on gravitational mass and gravitational mass on inertial mass, respectively. Their form suggests that these formulas, in addition to depending on the mass and size of the Universe, as expected, also depend on the magnitude of the force acting on the body.

Let's find a formula for Newton's second law, taking into account the obtained dependence (17). To do this, we substitute (17) into (1):

$$F = m_{in} a = m_G a \frac{FR^2}{FR^2 - \gamma M \cdot m_G} \quad (19)$$

$$F^2 R^2 - F\gamma M m_G = m_G a FR^2 \quad (20)$$

$$FR^2 = m_G a R^2 + \gamma M m_G \quad (21)$$

$$F = m_G a + \gamma \frac{M \cdot m_G}{R^2} = m_G \left(a + \gamma \frac{M}{R^2} \right) \quad (22)$$

Equating equalities (1) and (22), we obtain:

$$m_{in} a = m_G a + \gamma \frac{M \cdot m_G}{R^2} \quad (23)$$

$$m_{in} = m_G \left(1 + \gamma \frac{M}{aR^2} \right) \quad (24)$$

Formula (24) shows the relationship between inertial mass and a body's acceleration and gravitational mass. A body's acceleration a also depends on its location. Since we live on planet Earth, the fall of bodies is caused by the Earth's gravitational forces. Therefore, their acceleration is the acceleration due to gravity $a = g \approx 9{,}81 \approx 10\, m/s^2$. The desired formula for the relationship between inertial mass and gravitational mass is:

$$m_{in} = m_G \left(1 + \gamma \frac{M}{gR^2} \right) \quad (25)$$

Substituting the known data $\gamma = 6{,}67259 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$, $M \approx 10^{52}\, kg$, $R \approx 10^{27}\, m$ [13,14] into formula (25), we obtain a more specific version of this formula for our planet Earth:

$$m_{in} = m_G \left(1 + 6{,}67259 \cdot 10^{-14} \right) kg \,. \quad (26)$$

Considering that the acceleration of gravity [13]:

$$g = \gamma \frac{M_ç}{r_ç^2} \,, \quad (27)$$

And substituting (27) into (25), we obtain a version of the dependence of the inertial mass on the gravitational mass and size of the Earth:

$$m_{in} = m_G \left(1 + \frac{M}{M_ç} \cdot \frac{R_ç^2}{R^2} \right), \quad (28)$$

where $M_з$ and are $R_з$ – the mass and radius of the Earth, respectively. To find a similar relationship for other planets, it is sufficient to substitute the corresponding values for the acceleration due to gravity into formula (25) or the values of their mass and radius into formula (28). In some places in deep space, the magnitude of the acceleration due to gravity may be close to zero (a = 0), so the expression in parentheses in formula (24) will tend to infinity), but in this case, the gravimass of the body will tend to zero (since in these places the gravitational field itself will be close to zero). Therefore, the magnitude of the inertial mass of the body will tend to the corresponding limit.

Formulas (24–28) allow us to answer the question of the dependence of a body's gravitational mass on its velocity. Since these formulas do not involve the body's velocity anywhere, they are, by default, applicable not only to a moving body but also to a body at rest, and the inertial mass and gravitational mass used in them can also be rest masses. The body's velocity is equal to v = 0, although the body's acceleration is not zero a ≠ 0. Therefore, these formulas can also be applied to the moment of initiation (the beginning of the body's motion). In these formulas, gravitational mass is an argument, and inertial mass is a function. Indeed, inertial mass can only arise when gravitational mass already exists and has manifested itself. Now let us assume that the gravitational mass of a body m_G^v depends on the body's velocity according to the formula:

$$m_G^v = m_G f(v) \,, \quad (29)$$

where the function $f(v)$ is not yet known. Let the body begin to move with velocity v. Then the inertial mass of the body, in accordance with (24) and the special theory of relativity, will be equal to:

$$m_{in}^v = \frac{m_{in}}{\sqrt{1 - \frac{v^2}{c^2}}} = m_G^v \left(1 + \gamma \frac{M}{aR^2} \right) \quad (30)$$

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Substituting (29) into (30), we obtain:

$$\frac{m_{in}}{\sqrt{1-\frac{v^2}{c^2}}} = m_G\left(1+\gamma\frac{M}{aR^2}\right)f(v) \quad (31)$$

And substituting m_{in} from (24) into (31), we find the form of the function

$$f(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, \quad (32)$$

so that the gravitational mass depends on the speed of the body in the same way as the inertial mass.

$$m_G^v = \frac{m_G^0}{\sqrt{1-\frac{v^2}{c^2}}}, \quad (33)$$

where is m_G^0 – the rest gravitational mass.

If this function were different and, in particular, equal to one, which would mean the independence of the gravitational mass from the speed of the body, then a contradiction would arise with the identified interdependence of the gravitational mass and the inertial mass.

Conclusion

Formulas (24–28), as well as the available experimental data [2,13,15], confirm with accuracy the 10^{-14} Einstein Equivalence Principle (EPE), although there is a small but significant difference between inertial and gravitational masses, taking into account the dependence of the difference in these masses on the location of bodies in the Universe. The fact of the equivalence of inertial and gravitational masses can be seen in the reality of inertial forces. The French engineer J.V. Poncelet [16] understood the "force of inertia" as the real counterforce that a moving body exerts on connections or moving bodies. Such a force is, for example, the centrifugal force that pulls a rope attached to a mass performing circular motion. When we sit in an aeroplane accelerating for takeoff, our body is pressed with force into the seat. By accelerating our body, the aeroplane, through the seat, acts on it with force (1). Since every action is met by an equal and opposite reaction, our body, in turn, acts on the seat with an equal force. Thus, Poncelet's inertial forces can only be real in the case of interacting bodies. In general, one of the interacting bodies can always be some field of the physical vacuum.

In the works [17,18], the concept of *a fundamental field was introduced*, also called *a substantial field*, physical vacuum [18,19]. The fundamental field, disturbed by the absolute motion of bodies, is *an inertial* field, and the force of resistance of the field of the physical vacuum is *the force of inertia*. Inertial fields can include not only the field arising from translational forces, but also from rotational and torsional forces (*torsion fields)* [19]. D'Alembert's law (2) is formally equivalent to Newton's second law; it explains the same dependence of a body's acceleration on its mass and the force acting on it, but in terms of inertial force rather than a real external force. The inertial force arising in a fundamental (or any other) field, from the absolute motion of a body within it, motion with acceleration, leads to fluctuations in this field, which are perceived as an inertial mass. It has been proposed to call such virtual vacuum excitations *inertial masses.* [20]. They are quanta of the inertial field; the conditions for generating them in large quantities are described in [18]. Inertions, like any moving particles, are associated with wave motions, in this case with inertial waves. And since the rest mass of quanta of the inertial field is zero, their speed of motion must be equal to the speed of propagation of the corresponding inertial waves. Inertions, finding themselves in the Higgs field, filling the entire Universe [21], acquire a corresponding inertial mass, which, after interacting with the surrounding gravitational field of the Universe, will acquire weight and itself will become the source of the corresponding gravitational field.

In works [22,23], it was shown that at a body speed above $v = \omega \geq 235696.8871$, km/s a phase transition of matter into a massless state occurs, and then into a state with negative mass. If the temperature of a body becomes higher than a certain critical value $T \geq 2.17\cdot10^{36} m_0\,\mathrm{K}$, where m_0 – the rest mass of the body is in grams, depending on its chemical composition, then the mass of the body also becomes negative [24]. From the above-described interdependence of inertial and gravitational masses, it follows that the phase transition to a state with negative mass occurs for both types of mass, which corresponds to Einstein's Equivalence Principle (EPE). This principle facilitates the solution of many problems [25], in particular, the ability to determine the nature of the interactions of bodies with positive and negative mass [26].

One possible explanation for the dependence of gravitational mass and inertial mass on velocity is the hypothesis of G.P. Malkovsky [27], who believes that total mass m_v and rest mass m_0 express the quantity of matter. The law of the dependence of mass on velocity always applies not only to a moving body, but also to the field associated with that body. If the total mass of a body is understood as an amount of matter, then the velocity of a particle will not increase the amount of matter in the body, but rather the amount of matter in the field associated with the moving body. A similar position is developed in his theory of field physics by O.N. Repchenko [28].

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