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Annals of Mathematics and Physics

MATHEMATICS AND PHYSICS GROUP

ANNALS OF Mathematics and Physics OPEN ACCESS JOURNAL

ISSN: 2689-7636 DOI: https://dx.doi.org/10.17352/amp

Mini Review

How to Solve Convolution–Type Functional Equations?

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Received: 11 August, 2025
Accepted: 12 September, 2025
Published: 13 September, 2025

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Keywords: Variety; Spectral synthesis

Abstract

Convolution–type functional equations appear in all fields of pure and applied mathematics. The description of the solution space of such equations is based on the study of the fundamental problems of spectral analysis and spectral synthesis. Here we exhibit the possibility of using our recent results on spectral synthesis to offer a general method to solve systems of convolution–type functional equations.

AMS (2000) Subject Classification: 43A45, 22D99.

1. Introduction

A convolution-type functional equation is of the form

$$\mu * f = 0. \tag{1}$$

Clearly, we have to clarify the exact meaning of the symbols appearing in this equation. For instance, f may be a complex valued L^1-function on the reals R, and μ the Lebesgue measure on R, further * denotes the usual convolution:

$$\mu * f(x) = \int f(x-y)\,d\mu(y). \tag{2}$$

If μ is given, then we may look for all L^1-functions f satisfying (1): the set of all those functions will be called the *solution space*, or simply the *solution* of (1). On the other hand, we may interprete (1) in the following way: f is given, and we are looking for those measures μ satisfying equation (1). Obviously, we may change the possible domain of μ 's and f 's in equation (1) in any way such that the convolution of μ and f makes sense. Naturally, we may consider systems of such equations. In this work our setting will be as follows: given a commutative topological group G and $C(G)$ will denote the space of all continuous complex valued functions on G. Then the function f in (1) is supposed to be in $C(G)$. In order that the convolution makes sense we assume that μ is a compactly supported complex Borel measure – the set of all such measures will be denoted by $M_c(G)$. It is known that the space $C(G)$ is a locally convex topological vector space, if it is equipped with the linear operations (addition and multiplication by scalars), further with the topology of uniform convergence on compact sets (the compact-open topology). The advantage of this setting is that the topological dual of $C(G)$ can be identified with the space $M_c(G)$, moreover, if this latter space $M_c(G)$ is equipped with the weak*-topology, then its dual can be identified with $C(G)$. The pairing between $M_c(G)$ and $C(G)$ is given by

$$\langle \mu, f\rangle = \int f(x)\,d\mu(x).$$

Another important property of $M_c(G)$ is that the *convolution of measures* is defined by

$$\langle \mu * \nu, f\rangle = \int f(x+y)\,d\mu(x)\,d\nu(y) \tag{3}$$

for each μ, ν in $M_c(G)$ and for every f in $C(G)$. This multiplication, together with the linear operations on measures, makes $M_c(G)$ a commutative algebra – which is, in fact, a topological algebra under the weak*-topology. We call this algebra the *measure algebra* of G. As the convolution between $M_c(G)$ and $C(G)$ is also defined by the formula

Citation: Székelyhidi L. How to Solve Convolution–Type Functional Equations?. Ann Math Phys. 2025;8(5):160-165.
Available from: https://dx.doi.org/10.17352/amp.000160

$$\mu * f(x) = \int f(x-y)\,d\mu(y),$$

it turns *C(G)* into a *topological module* over the measure algebra. For more about the functional analysis topological group theoretical background the reader should consult with [1-3].

Summarizing, given a commutative topological group *G* and a set Γ of measures in $M_c(G)$, a system of convolution-type equations has the form

$$\mu * f = 0, \text{lcm} \mu \in \Gamma. \tag{4}$$

The function *f* in *C(G)* is called a *solution* of (4), if (4) holds for each *μ* in Γ. The *solution space* of (4) is the set of all solutions of (4). We note that the zero function is always a solution of a convolution–type system – it is called the *trivial solution*.

If Γ is given, then clearly, the solution space V_Γ of the system (4) is a closed linear space in *C(G)*, which is also *translation invariant*: if *f* is a solution, then the function $\tau_y f$, the *translate of f by y*, defined by

$$\tau_y f(x) = f(x+y)$$

is a solution as well, for every *y* in *G*. Observe, that $\tau_y f = \delta_{-y} * f$ where, in general, δ_y denotes the point mass at *y*. Hence translation invariance of a closed linear subspace *V* in *C(G)* is equivalent to convolution invariance: if *f* is in *V*, then *μ* * *f* is in *V*, for each *μ* in $M_c(G)$. In other words, translation invariance of a closed linear subspace in *C(G)* means that it is a closed submodule, which we shall call a *variety*. It follows that the solution space of a system of convolution–type equations is a variety.

Given the set Γ in $M_c(G)$ it is easy to see, that the solution space of (4) is the same as of the system, where Γ is replaced by the closed ideal in $M_c(G)$ generated by Γ. This means that, when studying the solution space of a system of convolution–type equations we may always suppose that Γ is a closed ideal in $M_c(G)$. This closed ideal is, in fact, the *annihilator v* of the variety *V*, which is the solution space of (4). And the dual concept is the annihilator of an ideal *I* in $M_c(G)$: it is the set *I* of all functions which are annihilated by all measures in the ideal *I*. We note that we obviously can use the concept of annihilator for any subset in *C(G)*, resp. $M_c(G)$. For our later purposes we recall a basic result about this variety-ideal annihilator correspondence:

Theorem 1

1. The annihilator of each subset in *C(G)* is a closed ideal, and the annihilator of each subset in $M_c(G)$ is a variety.

2. For each variety *V* and for every closed ideal *I* we have

$$V = V, I = I.$$

3. For every family of varieties V_α, and for every family of ideals I_α we have

$$\text{Ann}\,(\sum_\alpha V_\alpha) = \bigcap_\alpha \text{Ann}\, V_\alpha, \text{Ann}\,(\sum_\alpha I_\alpha) = \bigcap_\alpha \text{Ann}\, I_\alpha$$
$$\text{Ann}\,(\bigcap_\alpha V_\alpha) = \sum_\alpha \text{Ann}\, V_\alpha, \text{Ann}\,(\bigcap_\alpha I_\alpha) = \sum_\alpha \text{Ann}\, I_\alpha$$

We note that in this theorem the – possibly infinite – sums denote the closure of the corresponding set consisting of all finite sums. For the proof of this theorem and for further facts about the annihilator correspondence we refer to [4].

Here we show a simple example for the above concepts which may illustrate how to apply these ideas to solve systems of convolution–type functional equations.

Let *G* be an arbitrary commutative topological group, *m* : *GC* a continuous function and *y* an arbitrary element in *G*. We introduce the measure

$$\Delta_{m;y} = \delta_{-y} - m(y)\delta_0.$$

Let Γ denote the set of all measures $\Delta_{m,y}$ with *y* in *G* and we consider the system of equations (4). Let M_m denote the closure of the ideal in $M_c(G)$ generated by Γ. If *f* is a solution of (4), then we have

$$f(x+y) = m(y)f(x)$$

for each *x*, *y* in *G*. With *x* = 0 we have $f(y) = f(0)m(y)$, hence

$$f(0)m(x+y) = f(0)m(x)m(y)$$

holds for each *x*, *y* in *G*. If *f*(0) = 0, then *f* = 0, the trivial solution. If $f(0) \neq 0$, then

$$m(x+y) = m(x)m(y) \tag{5}$$

And *f* = *f*(0)*m*. It follows, that if (4) has a nontrivial solution, then *m* satisfies equation (5). The nonzero continuous functions satisfying (5) are called *exponential* functions, or simply *exponentials* on *G*: they play a fundamental role in the theory of convolution–type functional equations.

Summarizing, if *m* is not an exponential in (4), then (4) has only trivial solutions, that is, the solution space is *V* = {0}. Accordingly, the annihilator, that is the set of all measures annihilating *V*, is the whole measure algebra $M_c(G)$.

On the other hand, if *m* is an exponential, then the solution space of (4) is $V = \{c \cdot m : c \in C\}$, the constant multiples of *m*, which is a one dimensional variety. It follows that its annihilator ideal is a closed maximal ideal, which will be denoted by M_m. These ideals and their powers play a basic role in our investigation, as they are connected with some important function classes which serve as building blocks of the solution spaces of systems of convolution–type functional equations. The measure $\Delta_{m;y}$ is called the *modified difference*, and for *m* = 1 it is called simply *difference* and we write Δ_y for $\Delta_{1;y}$. The corresponding convolution operators

$$f \mapsto \Delta_{m;y} * f$$

are called *modified difference operators*, resp. *difference operators*

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if $m = 1$. We shall use the iterates of these operators: we write $\Delta_{m;y_1,y_2,\dots,y_n}$ for

$$\Delta_{m;y_1} * \Delta_{m;y_2} * \dots * \Delta_{m;y_n}.$$

In the case $m = 1$ we use the simpler notation $\Delta_{y_1,y_2,\dots,y_n}$. If $y_1 = y_2 = \dots = y_n = y$, then we write $\Delta^n_{m;y}$, resp. Δ^n_y if $m = 1$. For more about modified difference operators see [3-5].

2. Basic function classes

The example considered in the previous section can be generalized. We may consider any variety V in $C(G)$ – it is always the solution space of a system of convolution–type functional equations, namely, of the following one:

$$\mu * f = 0 1cm \mu \in V.$$

If V is $M_c(G)$, then the solution space is {0}. If, however, V is a proper ideal, then it is included in a maximal ideal M. If M is a closed maximal ideal, then M is a nonzero subvariety of V, and, by the maximality of M, it can be shown that M is one dimensional. It is very easy to see that in this case $M = M_m$ with some exponential m.

What happens, if every maximal ideal, which includes V is non-closed? Clearly, if a maximal ideal is non-closed, then it is dense, and in this case V includes no exponential: the corresponding system of convolution–type functional equations has no exponential solution. This case is somewhat pathological: on some very large discrete abelian groups there are systems of convolution–type functional equations having no exponential solution. The exact characterization of these groups is given in [6].

Observe, that the exponentials are exactly the eigenfunctions of all translation operators, or more generally, of all convolution operators of the form $f \mapsto \mu * f$, where μ is in the measure algebra. We can express this property by saying that the intersection of the kernel of all modified difference operators corresponding to m is the space of constant multiples of m. It is quite natural to ask about the kernel of the powers of modified difference operators, i. e. about the solution space of the system

$$\Delta_{m;y_1,y_2,\dots,y_{n+1}} * f = 0 \tag{6}$$

for each $y_1, y_2, \dots, y_{n+1}$ in G, where m is a given exponential and n is a natural number. Clearly, the solution space of this system is M^{n+1}_m. In the case $m = 1$ the functional equation (6) is the so-called *Fréchet equation* (see [1,7]), the solutions of which are called *generalized polynomials* of degree at most n. The case $n = 1$ is related to the classical *Cauchy functional equation*

$$f(x+y) = f(x) + f(y), \tag{7}$$

which implies

$$\Delta_{y_1,y_2} * f = 0 \tag{8}$$

with the additional property $f(0) = 0$. These functions are the so-called *additive functions* – they are actually homogeneous generalized polynomials of first degree. The general solution of (8) is of the form $f = a+c$, where a is additive and c is a complex number. These functions are called *linear functions*.

Observe, that the set of all linear functions is exactly the variety M_1^2. Indeed, f is the solution of (8) if and only if it is annihilated by all elements of $M_1 \cdot M_1$, which means

$$f(x+y_1+y_2) - f(x+y_1) - f(x+y_2) + f(x) = 0$$

for each x, y_1, y_2 in G. With $x = 0$ we get

$$f(y_1+y_2) - f(y_1) - f(y_2) + f(0) = 0,$$

which can be written as

$$f(y_1+y_2) - f(0) = f(y_1) - f(0) + f(y_2) - f(0),$$

that is, $f - f(0)$ is additive.

Going back to (6), its solutions are called *generalized m - exponential monomials* of degree at most n. By a simple calculation one can verify that f is an m - exponential monomial if and only if it is a generalized polynomial multiplied by m: $f = p \cdot m$, where p is a generalized polynomial. Of course, the degree of f is equal to the degree of p.

It is obvious, that linear combinations of generalized polynomials are generalized polynomials, but what about the linear combinations of m-exponential monomials with different m's? What happens if f is annihilated by $M_{m_1} \cdot M_{m_2}$ with different m_1 and m_2? The answer is given by the following theorem:

Theorem 2 *The continuous function $f : G \to C$ can be written in the form*

$$f = p_1 \cdot m_1 + p_2 \cdot m_2 + \dots + p_k \cdot m_k \tag{9}$$

where the p_j's are generalized polynomials and the m_j's are different exponentials, if and only if f is in $(M^{n_1+1}_{m_1} \cdot M^{n_2+1}_{m_2} \cdots M^{n_k+1}_{m_k})$ where $n_1, n_2, \dots, n_k$ are natural numbers.

Proof. Assume first that f has the given form (9). We have seen above that the annihilator of the functions of the form $p \cdot m$, where p is a generalized polynomial of degree at most n, is M^n. Using statement 3. in Theorem 1, we get that f is annihilated by $M^{n_1+1}_{m_1} \cap M^{n_2+1}_{m_2} \cap \dots \cap M^{n_k+1}_{m_k}$, where $n_1, n_2, \dots, n_k$ are the degrees of the generalized polynomials in (9). As the powers of different maximal ideals are co-prime, and the intersection of co-prime ideals is equal to their product (see e.g. [8]), the necessity part of the theorem follows.

Citation: Székelyhidi L. How to Solve Convolution–Type Functional Equations?. Ann Math Phys. 2025;8(5):160-165.
Available from: https://dx.doi.org/10.17352/amp.000160

The converse statement follows exactly by the same argument (see also [3]).

Functions f of the form (9) are called *generalized exponential polynomials*. Here the degree can be defined as the multi-index $(n_1, n_2, \ldots, n_k)$.

A natural question arises in connection with these function classes we have introduced: why do we use the adjective "generalized"? In fact, the class of generalized polynomials has a subclass, which is more important from the point of view of systems of convolution–type functional equations: a generalized polynomial is called a *polynomial*, if its variety is finite dimensional. In general, the *variety of the function f* in $C(G)$ is the smallest variety containing f: it is the intersection of all varieties containing f, and it is denoted by $\tau(f)$. For instance, every nonzero additive function a is a polynomial of degree 1, as $\tau(a)$ is the two dimensional space generated by 1 and a. However, if $G = Z^{\omega}$, the direct sum of countable many copies of the integers, then there are genralized polynomials on G, which are not polynomials. Roughly speaking, G is the set of all infinite sequences of integers, with only finitely many nonzero terms. A simple example for a generalized polynomial, which is not a polynomial is the following: let $a_i : G \to Z$ denote the $_i$-th projection of G, defined by

$$a_i(x) = x_i$$

for every x in G and for each i in R, then clearly, a_i is an additive function. We define

$$B(x, y) = \sum_i a_i(x) a_i(y)$$

for every x, y in G. The sum is finite for every x, y in G, and it is obvious that B is additive in both variables. We let

$$f(x) = B(x, x),$$

then it is easy to check that

$$\Delta_{y_1, y_2, y_3} * f = 0$$

for every y_1, y_2, y_3 in G, hence f is a generalized polynomial. On the other hand, a simple calculation shows that $\tau(f)$ is generated by the functions $1, a_i, f$ for i in R. As the functions a_i are linearly independent, hence $\tau(f)$ is of infinite dimension (see [9]).

Having introduced the concept of "polynomial", we also omit the adjective "generalized" from "generalized exponential monomial" and "generalized exponential polynomial", if the corresponding variety is finite dimensional. It follows that polynomials, exponential monomials and exponential polynomials on G have a nice description, as it is presented in the following result:

Theorem 3 *Let G be a commutative topological group.*

1. The function $f : G \to C$ is a polynomial of degree n if and only if there exist lineraly independent additive functions $a_1, a_2, \ldots, a_k$ and a complex polynomial $p : C^k \to C$ such that

$$f(x) = P((a_1(x), a_2(x), \ldots, a_k(x)), x \in G$$

2. The function $f : G \to C$ is an exponential monomial if and only if there exists an exponential $m : G \to C$ and a polynomial $p : G \to$ such that $f = p \cdot m$.

3. The function $f: G \to C$ is an exponential polynomial if and only if there exist different exponentials $m_1, m_2, \ldots, m_k : G \to C$ and polynomials $p_1, p_2, \ldots, p_k : G \to C$ such that $f = \sum_{I=1}^{k} p_i \cdot m_i$. If the p_i's are nonzero, then this representation of f is unique.

The basic property of exponential polynomials is expressed by the following theorem (see [10, Corollary 11], [3, Theorem 12.31]).

Theorem 4 *Let G be a commutative topological group and $f : G \to C$ a continuous function. The variety of f is finite dimensional if and only if f is an exponential polynomial.*

Proof. Here we prove the sufficiency – for the proof of the necessity see [10, Corollary 11]. Clearly, it is enough to show that if f is a polynomial, then $\tau(f)$ is finite dimensional. Indeed, if p is a polynomial and m is an exponential, then a simple calculation shows that ϕ is in $\tau(p)$ if and only if $\phi \cdot m$ is in $\tau(p \cdot m)$. On the other hand, the variety of the exponential polynomial

$$f = p_1 \cdot m_1 + p_2 \cdot m_2 + \cdots + p_k \cdot m_k$$

is included in the sum of the varieties $\tau(p_i \cdot m_i)$ for $i = 1, 2, \ldots, k$.

On the other hand, it is easy to check that if the polynomial p has the form

$$p(x) = P(a_1(x), a_2(x), \ldots, a_k(x)),$$

where p is a complex polynomial in k variables, and $a_1, a_2, \ldots, a_k$ are additive functions, then $\tau(p)$ is generated by the finitely many polynomials $x \mapsto \partial^{\alpha} P(a_1(x), a_2(x), \ldots, a_k(x))$, where α is a multi-index in N^k.

We shall see that the characteristic property of exponential polynomials generating finite dimensional varieties is of utmost importance in solving systems of convolution–type functional equations.

3. Spectral analysis and synthesis

In his fundamental paper [12] in 1947, Laurent Schwartz proved the following theorem:

Theorem 5 *Given any continuous complex valued function on the reals, all exponential polynomials in its variety span a dense subspace.*

To understand the importance of this result we observe that if the variety of the function f is $C(\mathbb{R})$, then the statement is obvious: by the Stone-Weierstrass theorem even the

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polynomials form a dense subspace in $C(\mathbb{R})$. The interesting case is the one where the variety of f is not the whole space $C(\mathbb{R})$: in this case f is called a *mean periodic function*. The above theorem says that every mean periodic function f can uniformly be approximated on compact sets by exponential polynomials, which satisfy every system of convolution–type equations, which is satisfied by f. Roughly speaking, if we know all exponential polynomial solutions of a system of convolution–type equations, then we know all solutions of that system.

It is quite reasonable to ask if this property holds on more general commutative topological groups. If we have a look at the example above, where we presented a generalized polynomial which is not a polynomial, then we can see that the variety of the function f defined by

$$f(x) = B(x,x)$$

for x in Z^{ω} is a counterexample. Indeed, we established that the variety of f is generated by the functions 1, the additive functions a_i for i in Z, and the function f itself, which is not a polynomial. The only exponential polynomials in this variety are linear functions, and it is obvious, that limits of linear functions are linear functions as well, hence f cannot be the limit of polynomials, which are in the variety. In the paper [13] the authors showed that if G is a discrete abelian group and there are infinitely many linearly independent additive function on G, then there are varieties on G for which the statement of Schwartz's theorem does not hold.

In order to dig deeper we introduce some definitions. We always assume that G is a commutative topological group. Our first observation is that if V is a variety on G and a generalized exponential polynomial is in V, then all the exponentials from which it is built up belong to V, as well. However, we mentioned above that there are commutative groups such that some systems of convolutio–type equations do not have exponential solutions. In other words, it may happen that a variety does not include any exponential: of course, in this case the exponential polynomials cannot span a dense subspace. We shall say that *spectral analysis holds for a variety* V, if every nonzero subvariety of V includes an exponential. We say that *spectral analysis holds on the group* G, if spectral analysis holds for each nonzero variety on G. This is equivalent to the property that every maximal ideal in the measure algebra is closed. We say that a variety V on G is *synthesizable*, if the exponential monomials in V span a dense subspace of V. We say that *spectral synthesis holds for* V, if every subvariety of V is synthesizable. Finally, we say that *the group* G *is synthesizable*, or *spectral synthesis holds on* G, if every variety on G is synthesizable. Clearly, spectral synthesis for a variety implies spectral synthesis for it, but the converse is not true: in [6] the authors proved that on a discrete abelian group spectral analysis holds if and only if the torsion-free rank of the group is less than the continuum, and in [13] it has been proved that on a discrete abelian group spectral synthesis holds if and only if the torsion-free rank of the group is finite.

In the non-discrete case the situation is more complicated. The first natural question is whether Schwartz's result can be extended for functions in several variables – in other words, does spectral synthesis hold on n for $n > 1$? Somewhat surprisingly, the answer is negative. In fact, in [14] the author presents two counterexamples in $\mathbb{R}^2$. In the first case a system of convolution–type equations – consisting of two equations – is given such that the exponential monomials do not span a dense subspace in the solution space. In the second case another system of six convolution–type equations is presented such that the solution space is nontrivial, but the system has no exponential solution. In the light of these negative results the most interesting question arises: how to characterize those commutative topological groups having spectral synthesis?

In our recent work [15] we introduced a method, called *localization of ideals* in the Fourier algebra of a locally compact abelian group. The main idea is to consider *differential oparators* on the Fourier algebra, which are polynomials of *first order derivations*. Given an ideal we say that it is *localizable*, if it has the following property: if a function is annihilated by all differential operators, which annihilate the ideal, then this function belongs to the ideal, as well. The ideals of the Fourier algebra correspond in a one-to-one way to the ideals of the measure algebra, hence this localizability concept can be applied for the ideals of the measure algebra. Our main result in this respect is that a closed ideal of the measure algebra is localizable if and only if its annihilator variety is synthesizable. This simple criteria for synthesizability leads to a complete characterization of those locally compact abelian groups having spectral synthesis in the following two results (see [15]).

Theorem 6 *The compactly generated locally compact abelian group G is synthesizable if and only if it is topologically isomorphic to* $R^a \times Z^b \times C$, *where* a,b *are nonnegative integers with* $a \leq 1$, *and C is a compact abelian group.*

In the next theorem B denotes the closed subgroup of compact elements in the group G: those elements, which generate a compact subgroup.

Theorem 7 *The locally compact abelian group G is synthesizable if and only if G/B is topologically isomorphic to* $R^a \times Z^b \times D$, *where a, b are nonnegative integers with* $a \leq 1$, *and D is a discrete abelian group of finite rank.*

Conclusion

Now we can offer a method for solving systems of convolution–type functional equations on locally compact abelian groups. Given the system (4) first we find its exponential solutions: these exponentials m form the *spectrum* of the system. These are the common roots of the Fourier transforms of the measures μ in Γ. The next step is to find the "multiplicities" of these roots: these are realized by the exponential monomials $p \cdot m$ corresponding to the spectrum, and they form the *spectral set* of the system. Here p is a polynomial, which can be written in the form

$$p(x) = P(a_1(x), a_2(x), \ldots, a_k(x)),$$

where the additive functions are supposed to be linearly independent, and p is a complex polynomial in k variables. Substituting $p \cdot m$ into the system we obtain

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$$(\mu * pm)(x) = \int p(x-y)m(x-y)d\mu(y) = 0$$

for each x in G and for every μ in Γ. To find the polynomial solutions of this system is a purely algebraic job. By the above theorems, if G satisfies the assumptions, then the solution space of the system consists of the limits of convergent sequences formed by the elements of the spectral set. We note that, if G does not satisfy the conditions of the above theorems, still there is a possibility that spectral synthesis holds for the given system – it depends on the localizability of the ideal in question.

For more about the history, classical problems and results related to spectral analysis and synthesis see e.g. [16-23].

Acknowledgement

The research was supported by the the Hungarian National Foundation for Scientic Research (OTKA), Grant No. K-134191.

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Citation: Székelyhidi L. How to Solve Convolution–Type Functional Equations?. Ann Math Phys. 2025;8(5):160-165.
Available from: https://dx.doi.org/10.17352/amp.000160