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Research Article

# Miraj's Numo: Generalized Algebraic Identity for the Sum and Difference of Powers

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## Abstract

This paper is concerned with the presentation of Miraj's Numo: a research algebraic identity that converts the sums and differences of two nth powers into differences of squares. The method hinges on cleverly defining a parameter m that depends upon the ratio of the powers so that the transformation works for any two real numbers a, b, and any integer exponent n. We present a stepwise derivation, along with thorough cases, to show that Miraj's Numo generalizes classic identities and contracts expressions to simpler forms. In the realm of mighty factors, the identity opens a new viewpoint to approach polynomial equations and factorization problems and may be of assistance in simplifying computations in higher algebra and number theory. With Miraj's Numo in hand-that looks into hidden structures within power sums-there blossoms a path towards applications in such research areas as computational algebra, and even theoretical physics, where such algebraic manipulations find their needed application.

## 1. Introduction

The sum of powers, $a^n + b^n$, has been a subject of interest in algebra for centuries. Various identities have been proposed to simplify such sums for specific powers, such as cubes and squares. However, these identities are often limited in scope and do not generalize well for all values of $n \in$ Z. My paper presents *Miraj's Numo*, as a generalized algebraic formula that expresses the sum of two nth powers as a difference of squares for all integer values of $n$  Z.

The significance of Miraj's Numo lies in its versatility and generalization. By introducing a custom parameter, $m_n$, which varies based on the value of $n$, we are able to express the sum of powers $a^n + b^n$ in a compact and elegant form. This identity not only unifies previous results for specific powers but also extends them to an infinite range of integer values for $n$, covering both positive and negative components [1,2].

But what's really new about *Miraj's Numo* is that you finally get a general way to express the sum or difference of two nth powers as a difference of squares, for each separate integer exponent $n$ - something old identities like Sophie Germain's or sum of cubes only did for special cases. Unlike traditional formulas confined to specific powers, *Miraj's Numo* breaks these boundaries, introducing a flexible parameter $m_n$ that adapts dynamically, enabling a unified, elegant, and broad-reaching identity. This not only deepens our algebraic understanding but opens new doors for applications in number theory, polynomial factorization, and computational algebra.

## 2. Derivation of Miraj's Numo

To derive *Miraj's Numo*, we begin with the following identity for the difference of squares:

$$x^2 - y^2 = (x+y)(x-y)$$

We aim to express $a^n + b^n$ and $a^n - b^n$ in a similar form. By strategically defining an auxiliary parameter $m_n$, we construct a difference of squares. This is the core idea behind Miraj's Numo.

$$a^n \pm b^n = \left(a\left(m_n + a^{n-2}\right)\right)^2 - \left(a\left(m_n - a^{n-2}\right)\right)^2$$



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Here, the parameter $m_n$ is key in adjusting the expression for both the sum and the difference of powers. The general form of the identity involves the term $a^{n-2}$, which helps adjust for the degree of the power, and $m_n$, which varies based on $n$.

For the sum of numbers $a^n + b^n$, we modify the formula as follows:

$$a^n + b^n = \left(a\left(m_n + a^{n-2}\right)\right)^2 - \left(a\left(m_n - a^{n-2}\right)\right)^2$$

For the difference of numbers $a^n - b^n$, we modify the formula as follows:

$$a^n - b^n = \left(a\left(m_n + a^{n-2}\right)\right)^2 - \left(a\left(m_n - a^{n-2}\right)\right)^2$$

Thus, $m_n$ as a parameter is same for both the sum and the difference cases. Now, we need to define $m_n$ properly.

### 2.1. Defining $m_n$ (Miraj's Change)

The key to generalizing Miraj's Numo is the definition of the parameter $m_n$. We define $m_n$ as:

$$m_n = \frac{1}{4}\left(1 \pm \left(\frac{b}{a}\right)^n\right)$$

Notice that in this formula, the ± sign applies between 1 and $\left(\frac{b}{a}\right)^n$, adjusting the formula for both the sum and the difference cases [3].

### 2.2 Simplifying the expression

Now, let's expand the squares on both sides of the difference of squares formula:

$$\left(a\left(m_n + a^{n-2}\right)\right)^2 = a^2\left(m_n^2 + 2m_n a^{n-2} + a^{2n-4}\right)$$

$$\left(a\left(m_n - a^{n-2}\right)\right)^2 = a^2\left(m_n^2 - 2m_n a^{n-2} + a^{2n-4}\right)$$

Now, subtract the second expansion from the first:

$$a^2\left(m_n^2 + 2m_n a^{n-2} + a^{2n-4}\right) - a^2\left(m_n^2 - 2m_n a^{n-2} + a^{2n-4}\right)$$

Simplifying the expression:

$$= a^2\left(4m_n a^{n-2}\right) = 4m_n a^n$$

Thus, for the identity to hold, we must have:

$$a^n + b^n = 4m_n a^n$$

And similarly for the difference case:

$$a^n - b^n = 4m_n a^n$$

### 2.3 Final form of Miraj's Numo

The final generalized formula for Miraj's Numo is:

$$a^n \pm b^n = \left(a\left(m_n + a^{n-2}\right)\right)^2 - \left(a\left(m_n - a^{n-2}\right)\right)^2$$

where the parameter $m_n$ is given by:

$$m_n = \frac{1}{4}\left(1 \pm \left(\frac{b}{a}\right)^n\right)$$

This holds for all $a, b \in \mathbb{R}, a \neq 0$, and $n \in Z$.

## 3. Power series of Miraj's identity

In this section, we present power families of Miraj's general identity, illustrating the sum and difference of powers expressed as a difference of squares. Each identity is named according to the exponent $n$, showcasing the elegant structure of the series.

### 3.1. Miraj's Cubo (*n* = 3)

$$a^3 \pm b^3 = \left(a(m + a)\right)^2 - \left(a(m - a)\right)^2, \quad \text{where} \quad m = \frac{1}{4}\left(1 \pm \left(\frac{b}{a}\right)^3\right)$$

### 3.2. Miraj's Quarto (*n* = 4)

$$a^4 \pm b^4 = \left(a\left(m + a^2\right)\right)^2 - \left(a\left(m - a^2\right)\right)^2, \quad \text{where} \quad m = \frac{1}{4}\left(1 \pm \left(\frac{b}{a}\right)^4\right)$$

### 3.3. Miraj's Quinto (*n* = 5)

$$a^5 \pm b^5 = \left(a\left(m + a^3\right)\right)^2 - \left(a\left(m - a^3\right)\right)^2, \quad \text{where} \quad m = \frac{1}{4}\left(1 \pm \left(\frac{b}{a}\right)^5\right)$$

### 3.4. Miraj's Sexto (*n* = 6)

$$a^6 \pm b^6 = \left(a\left(m + a^4\right)\right)^2 - \left(a\left(m - a^4\right)\right)^2, \quad \text{where} \quad m = \frac{1}{4}\left(1 \pm \left(\frac{b}{a}\right)^6\right)$$

### 3.5. Miraj's Septo (*n* = 7)

$$a^7 \pm b^7 = \left(a\left(m + a^5\right)\right)^2 - \left(a\left(m - a^5\right)\right)^2, \quad \text{where} \quad m = \frac{1}{4}\left(1 \pm \left(\frac{b}{a}\right)^7\right)$$

### 3.6. Miraj's Octo (*n* = 8)

$$a^8 \pm b^8 = \left(a\left(m + a^6\right)\right)^2 - \left(a\left(m - a^6\right)\right)^2, \quad \text{where} \quad m = \frac{1}{4}\left(1 \pm \left(\frac{b}{a}\right)^8\right)$$

### 3.7. Miraj's Nono (*n* = 9)

$$a^9 \pm b^9 = \left(a\left(m + a^7\right)\right)^2 - \left(a\left(m - a^7\right)\right)^2, \quad \text{where} \quad m = \frac{1}{4}\left(1 \pm \left(\frac{b}{a}\right)^9\right)$$

### 3.8. Miraj's Deco (*n* = 10)

$$a^{10} \pm b^{10} = \left(a\left(m + a^8\right)\right)^2 - \left(a\left(m - a^8\right)\right)^2, \quad \text{where} \quad m = \frac{1}{4}\left(1 \pm \left(\frac{b}{a}\right)^{10}\right)$$



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**Infinite extension of Miraj's identity:** Miraj's Identity is not limited to the powers exhibited here; in fact, the series continues for all integer powers $n \in Z$. The instances of Cubo ($n = 3$) through Deco ($n = 10$) are assigned names to allow for real-world applications and intuitive understanding. Beyond Deco, the pattern continues indefinitely with no less beauty or universality, and the naming convention can be continued as desired or replaced with numerical notation. This is consistent with the profound and limitless nature of Miraj's Numo, which can express any sum or difference of powers as a difference of squares [4,5].

# 4. Examples of Miraj's Numo

Examples: Sum of cubes example: Let $a = 2$, $b = 1$, and $n = 3$. We calculate the sum:

$$2^3 + 1^3 = 8 + 1 = 9.$$

Using Miraj's Cubo identity,

$$m = \frac{1}{4}\left(1 + \left(\frac{1}{2}\right)^3\right) = \frac{1}{4}\left(1 + \frac{1}{8}\right) = \frac{9}{32}.$$

Then,

$$\left(2(m+2)\right)^2 - \left(2(m-2)\right)^2 = (2\times\frac{65}{32})^2 - (2\times\frac{-55}{32})^2 = \left(\frac{130}{32}\right)^2 - \left(-\frac{110}{32}\right)^2 = 9,$$

confirming the sum identity.

**Difference of cubes example:** Let $a = 3$, $b = 1$, and $n = 3$. We calculate the difference:

$$3^3 - 1^3 = 27 - 1 = 26.$$

Using Miraj's Cubo identity,

$$m = \frac{1}{4}\left(1 - \left(\frac{1}{3}\right)^3\right) = \frac{1}{4}\left(1 - \frac{1}{27}\right) = \frac{26}{108} = \frac{13}{54}.$$

Then,

$$\left(3(m+3)\right)^2 - \left(3(m-3)\right)^2 = \left(3\times\frac{175}{54}\right)^2 - \left(3\times\frac{-149}{54}\right)^2 = 26,$$

confirming the difference identity.

# 5. Applications and future scope

## Symbolic computation and simplification

Miraj's Numo holds significant potential for symbolic algebra systems such as *Mathematica*, *Maple*, and *SymPy*, where power expressions frequently arise. The identity can be applied to:

- Introduce a new transformation rule for power expressions using difference-of-squares.
- Reduce algebraic complexity in symbolic simplification algorithms.
- Enable recursive or nested simplifications that enhance pattern recognition and substitution mechanisms.

## Control theory and polynomial systems

In areas like digital signal processing, control systems, and system modeling, polynomial expressions of powers naturally emerge. Potential applications of Miraj's Numo include:

- Simplifying transfer functions and polynomial expansions in Laplace and Z-domain analysis.
- Reducing symbolic overhead during algebraic manipulation of control equations.
- Supporting real-time modeling tools by offering an alternative square-based formulation.

## Further research directions

- Extending Miraj's Numo to rational, fractional, or even complex exponents.
- Exploring its role in finite field arithmetic, modular systems, and algebraic cryptography.
- Investigating recursive formulations and self-similar algebraic structures based on Numo.
- Studying connections with Faulhaber's Formula, Bernoulli polynomials, and power-sum expansions.

# 6. Conclusion

Miraj's Numo is a generalized identity for the sum of powers, offering a new way to express $a^n + b^n$ as a difference of squares for all integer values of $n$. This identity provides a simple, elegant form for the sum of powers, and it is valid for any integer $n \in Z$, from $-\infty$ to $\infty$. The derivation of the identity using the difference of squares method showcases its mathematical robustness and utility.

## Originality statement

*The identity presented in this paper, Miraj's Numo, is an original contribution by the author. It introduces a new algebraic formulation for expressing the sum and difference of powers as a difference of squares, valid for all integer powers. This approach, including the parameterized structure involving* $m_n$*, is not derived from existing literature and represents the author's independent research and innovation. The work has not been published or submitted elsewhere.*

## Declarations

**Ethical statement:** This research did not involve any studies with human participants or animals. The work is purely theoretical in nature, focusing on original mathematical identities. All results were independently derived by the author, and no ethical approval was required.



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