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Short Communication

Simple Proof of the Hardy–littlewood Conjecture

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Abstract

The Hardy–Littlewood conjecture suggests that every odd integer $2n + 1$ greater than or equal to 7 is the sum of three prime numbers, two of which are equal. In this paper, we present a simple approach that attempts to prove this conjecture.

Introduction

In this paper, we attempt to prove a formulation related to the Hardy–Littlewood conjecture using a simplified approach. Our method proposes an inductive argument based on a structural reformulation of odd integers and draws parallels to the ternary Goldbach conjecture, which has already been proven using modern analytic techniques [1].

The Hardy–Littlewood conjecture, originally introduced in 1923 [2], concerns the representation of numbers as sums of primes and underpins major unsolved problems in additive number theory. More recent progress by Tao [1] and Chen and Wang [3] has advanced our understanding of such representations, particularly in the context of the Goldbach-type problems. However, a complete and elementary proof remains elusive for some versions of these conjectures. This paper attempts to construct a simplified version of such a proof.

Statement of the conjecture

We consider the following conjectural formulation:

Every odd number greater than or equal to 7 can be written in the form:

$$2n + 1 = 2p + q,\ n \in N,\ n \geq 3,$$

where p and q are prime numbers.

This format suggests that every such number can be expressed as the sum of two prime-related components, linking it to aspects of the Hardy–Littlewood and Goldbach conjectures [1,3].

Auxiliary conjecture

To proceed with the inductive approach, we introduce an auxiliary conjecture:

For every pair of odd prime numbers p and q there exists a prime number $r \leq p$ such that

$$2(p - r + 1) + q \text{ is also a prime number.}$$

Although this is a new conjecture without formal justification, it conceptually draws from the heuristic foundations underlying prime distribution models like those described by Bateman and Horn [4].

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Attempted proof via mathematical induction

Base Case: n=3

We verify the base case:

7=2×2+3,

where both 2 and 3 are primes, consistent with the conjecture.

Inductive step

Assume that for some n=k, is true and show for n=k+1 is true

We have 2k+3=2k+1+2

We have 2k+1=2p+q and p,q are prime numbers after using the hypothesis of Induction

Now we use the theorem 1 then there exist a prime number $r \le p$ such as 2(p−r+1)+q

is prime number then

2k+3=2p+q+2=2r+2(p−r+1)+q =2r+s such as s is a prime number then we complete

the proof.

where both r and s are primes.

This supports the inductive progression of the conjecture.

While this inductive outline is structurally complete, it depends critically on the assumed auxiliary conjecture, whose validation would require deeper analytic or heuristic foundations, possibly related to density results like those discussed by Hardy and Littlewood [2] or Bateman and Horn [4-9].

Conclusion

We have proposed a simplified inductive framework to validate a form of the Hardy–Littlewood conjecture. However, this argument crucially relies on an auxiliary conjecture regarding prime behavior, which itself remains unproven. While the approach offers an intuitive pathway, further mathematical scrutiny and formal proof of the auxiliary conjecture are necessary.

References

1. Tao T. Every odd number greater than 1 is the sum of at most five primes. Math Comput. 2014;83(286):997–1038.
2. Hardy GH, Littlewood JE. Some problems of 'Partitio Numerorum.' III. On the expression of a number as a sum of primes. Acta Math. 1923;44:1–70. Available from: http://dx.doi.org/10.1007/BF02403921
3. Chen JR, Wang TZ. On odd Goldbach problem. Acta Math Sinica. 1989;32:702–718.
4. Aletheia-Zomlefer S, Fukshansky L, Garcia SR. The Bateman–Horn conjecture: Heuristic, history, and applications. Expo Math. 2020;38(4):430–479. Available from: https://doi.org/10.48550/arXiv.1807.08899
5. Helfgott HA. The ternary Goldbach conjecture is true. Ann Math Stud [Preprint]. 2015. Available from: https://arxiv.org/abs/1312.7748
6. Montgomery HL, Vaughan RC. Multiplicative number theory I: Classical theory. Cambridge: Cambridge University Press; 2007. Available from: https://assets.cambridge.org/97805218/49036/frontmatter/9780521849036_frontmatter.pdf
7. Granville A. Harald Cramér and the distribution of prime numbers. Scand Actuar J. 1995;1995(1):12–28. Available from: https://doi.org/10.1080/03461238.1995.10413946
8. Pomerance C. A tale of two sieves. Notices Am Math Soc. 1996;43(12):1473–1485. Available from: https://www.ams.org/notices/199612/pomerance.pdf
9. Nathanson MB. Additive number theory: The classical bases. New York: Springer; 1996. (Graduate Texts in Mathematics; vol. 164). Available from: https://download.e-bookshelf.de/download/0000/0017/29/L-G-0000001729-0002340718.pdf

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