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Short Communication

Critical Line in the Euler–Riemann Zeta Function

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Abstract

This study focused on the transformation of an exponentially growing divergent function $\sin(\text{Rln}(x))$ into a convergent function by its complementary exponential function x^i in such a manner that the sizes of positive and negative areas under $\sin$ would be the same. The transformation will provide the entire $\sin$ function with self-compensatory behavior. The exponent's value was computed and found to be $-1/2$, which is the only exponent, which lets entire product of the function converge to zero (sum of area for positive real numbers and sum of products for natural numbers). The exponent $-1/2$ is algebraically and geometrically inevitable for the function $x^i \sin(\text{Rln}(x))$ converging to zero. This result directly impacts the position of the critical line in the Euler-Riemann zeta function.

Introduction

The critical line was defined as the entire real section of the recognized non-trivial zeros for the Euler-Riemann zeta function with complex numbers equal to $-1/2$. All non-trivial zeros exist on the critical line, according to the Riemann hypothesis, which is regarded as one of mathematics' most challenging open problems, providing the study's motivation (Figure 1) [1–3].

Result and discussion

First, a helper harmonic periodic function was constructed above the $\sin(\text{Rln}(x))$ curve using simplification, which I named *linus* (Figure 2). The *linus* function was developed by subtracting $n\pi$, which is equivalent to $\arcsine(\sin(\text{Rln}(x)))$ function (which I termed *assRln(x)* function), where R is any real number over 10. This allowed *linus* values to remain within the range between an interval of $-\pi/2$ to $+\pi/2$.

I made further simplifications and created another helper harmonic periodic function, which I named *trianglus* sustaining, from straightforward right-angled triangles filled

under the *linus* function in order to facilitate precise area calculations under the *linus* curve (Figure 2).

Local minima m , which have a value of zero, are shared by all the functions. Local positive and negative maxima M have values of $\pm\pi/2$ for *linus* and *trianglus* and ± 1 for *sinus*.

It was assumed that the area ratio A to B would remain constant between the *sinus*, *linus*, and *triangle harmonic* periodic functions as well as in their harmonic derivatives produced by their respective exponentially complementary functions (for evidence read further).

In order to calculate local maxima and minima, I first defined the following formulas: local max $\sin(\text{Rln}(x)) = 1$, $\text{Rln}(x)$ must equal $(2n+1)\pi/2$; local min $\sin(\text{Rln}(x)) = 0$, $\text{Rln}(x)$ must equal $(2n)\pi/2$. As an illustration, consider the following: local maximum $M65 = \exp(65\pi/2R)$, local minimum $M66 = \exp(66\pi/2R)$, and local minimum $M67 = \exp(67\pi/2R)$. In conclusion, the distribution of the local maxima and minima is exponential.

Subsequently, I computed the triangular areas ratios B to A , which are determined by the local maxima $M65$ and $M67$

and equal $\exp(\pi/2R)$ (Figure 2). Similarly, C to A is equal to $\exp(2\pi/2R)$, and D to A is equal to $\exp(3\pi/2R)$. In conclusion, the areas in divergent periodic harmonic $\sin(R\ln(x))$ function grow exponentially.

At this point, using a complementary exponential function, The aim was to transform the divergent $\sin(R\ln(x))$ by complementary exponential function into convergent $\sin$

function, which would compensate for the exponential growth of the area in each periods and achieve a 1:1 ratio between area segments A and B.

The conditions were satisfied by the simple exponential function x^t , which is also coherent with the imaginary function in the Euler-Riemann zeta function with complex numbers (Figure 1). The exponent t for the A and B areas could be determined using the triangular simplification, and it was found to be equal to $-1/2$ (Figure 2).

Finally, I examined the x^t exponential complementary function on $\sin(R\ln(x))$ with an exponent of $-1/2$. As I had assumed, $\sin(R\ln(x))$ could be treated using the output of the harmonic periodic function $\sin$ and $\cos$. The area-ratio B to A under the $x^{-1/2}\sin(R\ln(x))$ curve is 1 (proven integral calculus) (Figure 4). Nevertheless, in the $x^{-1/2}\sin(R\ln(x))$ function, the areas A to C continue to rise exponentially by $\exp(2\pi/2R)$.

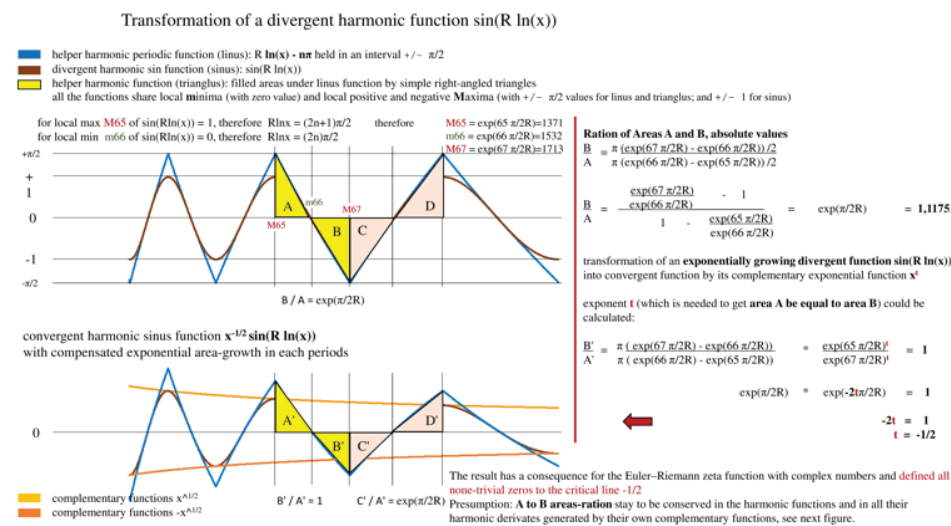
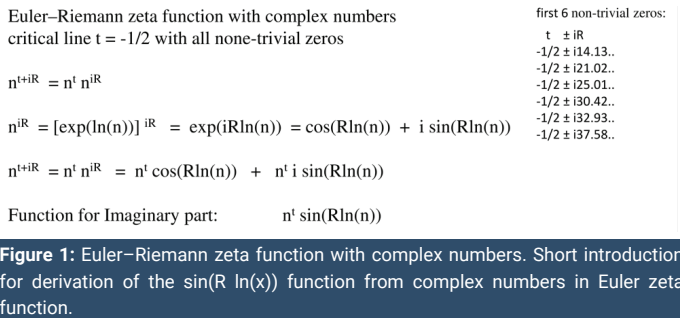


Figure 2: Transformation of a divergent harmonic function $\sin(R \ln(x))$. Transformation of an exponentially growing divergent but harmonic function $\sin(R \ln(x))$ into convergent function by its complementary exponential function x^t .

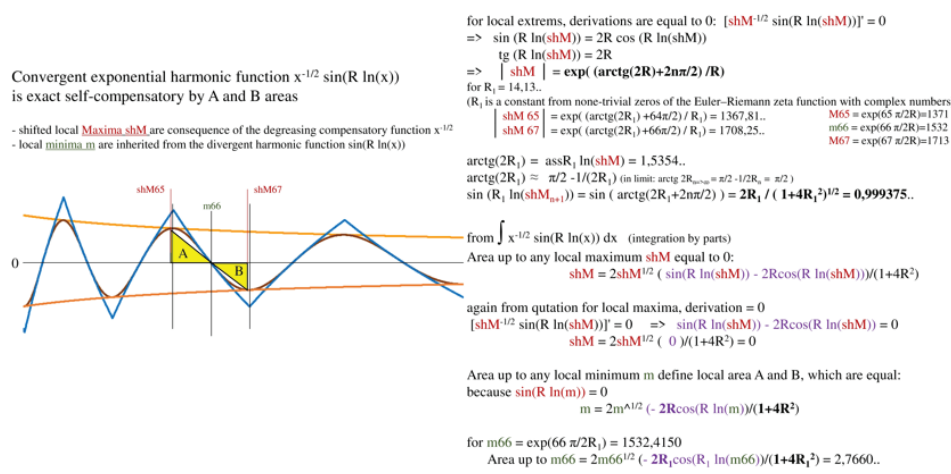


Figure 3: Self-compensatory property. Convergent exponential harmonic sin function $x^{-1/2} \sin(R \ln(x))$ has self-compensatory due its positive A and negative B areas.

The degreasing compensating function $x^{-1/2}$ induces deformation of the sin curve through shifted local maxima shM. These can be calculated from derivation $[x^{-1/2}\sin(R\ln(x))]'$ equal zero. That means that $\text{tg}(R\ln(\text{shM})) = 2R$ and $|\text{shM}| = \exp((\arctg 2R + 2n\pi/2) / R)$. As a result, the $2R$ value modifies the shape of the sinus curve $x^{-1/2}\sin(R\ln(x))$ by which defines a horizontal shift for the local maxima. Nonetheless, the local minima m are inherited from the divergent sin function $\sin(R\ln(x))$ (Figure 3).

Due to the compensatory function, the transformed function $x^{-1/2}\sin(R\ln(x))$ is a periodic harmonic function that converges to zero (values of R tested between 10–20; results for $R < 10$ were omitted, therefore not sure about validity for R below 10, data not included). This is only true for continuous functions, where the real numbers in the input are continuous. In contrast, an $n^{-1/2}\sin(R\ln(n))$ function, where n are natural numbers, the function becomes discontinuous, introducing imperfections in the otherwise harmonic functions $x^{-1/2}\sin(R\ln(x))$ and $x^{-1/2}\text{ass}(R\ln(x))$ (particularly with initial values in an intrinsically disordered region, IDR) (Figure 4).

The position of the sinus curve $n^{-1/2}\sin(R\ln(n))$ with regard to the natural numbers (the raster), influences how much volume is produced in the areas A and B under the sinus curve (strips formation under sin).

As a result, practically all of $n^{-1/2}\sin(R\ln(n))$ converge away from zero due to accumulated imperfections. The only functions that employ non-trivial zero constants R_x , accrue exactly the same volumes A and B under sinus including their imperfections, and converge to zero. The $2R_x$ define their shifted maxima; sinus shape deformation decreases as R value increases, whereas frequency increases as R value increases (Figure 3).

Regions A and B have almost identical volumes after the intrinsically disordered region (IDR), despite the fact that $n^{-1/2}\sin(R\ln(n))$ is still an exponentially expanding function and that the region A and B accommodate the exponentially growing counts of the natural numbers with their imperfections. Crucially, because of imperfection links to the natural numbers by both size and counts, the self-compensation effect applied on imperfection as well (Figure 4).

Outside of the IDR region, the $n^{-1/2}\sin(R\ln(n))$ continual area-summations to the local minima grow exponentially by $\exp(\pi/2R)$, while the continual area-summations to the local maxima decrease exponentially by $\exp(-\pi/2R)$ and importantly, approach zero at the local maxima. This behavior does not hold true for other exponents (Figure 5).

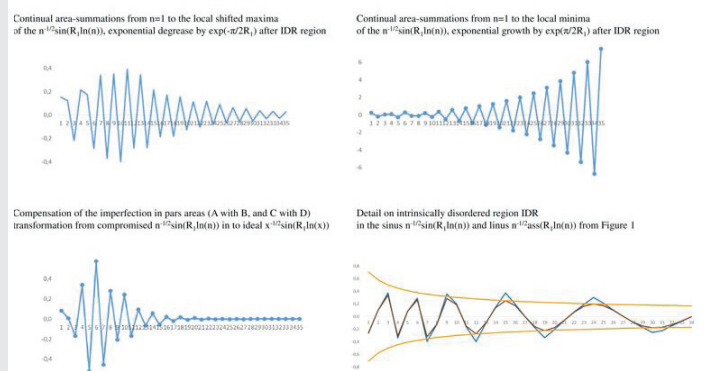


Figure 4: Continual area-summations from $n=1$ to the local shifted maxima of the $n^{-1/2}\sin(R, \ln(n))$.

Sums from $n=1$ up to shifted maximum shM84 for $n^{-1/2}\sin(R, \ln(n))$ function

t exponent	shM84	Sum n=1 to shM84	$[\text{shM}'\sin(R, \ln(\text{shM}))]'=0$ shifted maxima shM calculus	$\int \text{shM}'\sin(R, \ln(\text{shM}))dx$ area calculus upto shM
- 0.1	12652,0619	+19.8622	$-0.2\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM})) = 0$	$-\text{shM}^4 (-0.8\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM}))) (0.64+R^2) \neq 0$
- 0.2	12645,7315	+ 5.8895	$-0.3\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM})) = 0$	$-\text{shM}^4 (-0.6\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM}))) (0.36+R^2) \neq 0$
- 0.3	12639,4056	+ 1.5199	$-0.4\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM})) = 0$	$-\text{shM}^4 (-0.4\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM}))) (0.16+R^2) \neq 0$
- 0.4	12633,0848	+ 0.3118	$-0.5\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM})) = 0$	$-\text{shM}^4 (-0.2\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM}))) (0.04+R^2) \neq 0$
- 0.5	12626,7696	+ 0.0065	$-0.6\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM})) = 0$	$-\text{shM}^4 (-0.2\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM}))) (0.04+R^2) \neq 0$
- 0.6	12620,4607	- 0.0549	$-0.7\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM})) = 0$	$-\text{shM}^4 (-0.2\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM}))) (0.04+R^2) \neq 0$
- 0.7	12614,1588	- 0.0543	$-0.8\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM})) = 0$	$-\text{shM}^4 (-0.2\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM}))) (0.04+R^2) \neq 0$
- 0.8	12607,8645	- 0.0479	$-\sin(R, \ln(\text{shM})) + R\cos(R, \ln(\text{shM})) = 0$	
- 0.9	12601,5782	- 0.0446		

*real local shifted maxima correspond to reverse core values of the integral calculus:
 $\text{shM84}_{(n=0.6)} = \exp((84\pi/2 + \arctg(14.13/0.6))/14.13) = 12633.0848$
 $\Rightarrow 14.13 \cdot \cos(14.13 \cdot \ln(\text{shM84}_{(n=0.6)})) - 0.6 \cdot \sin(14.13 \cdot \ln(\text{shM84}_{(n=0.6)})) = 0$
 $\text{but } 14.13 \cdot \cos(14.13 \cdot \ln(\text{shM84}_{(n=0.6)})) - 0.6 \cdot \sin(14.13 \cdot \ln(\text{shM84}_{(n=0.6)})) = -0.199919964$
 $\text{shM84}_{(n=0.6)} = \exp((84\pi/2 + \arctg(14.13/0.6))/14.13) = 12620.4607$
 $\Rightarrow 14.13 \cdot \cos(14.13 \cdot \ln(\text{shM84}_{(n=0.6)})) - 0.6 \cdot \sin(14.13 \cdot \ln(\text{shM84}_{(n=0.6)})) = 0$
 $\text{but } 14.13 \cdot \cos(14.13 \cdot \ln(\text{shM84}_{(n=0.6)})) - 0.6 \cdot \sin(14.13 \cdot \ln(\text{shM84}_{(n=0.6)})) = 0.199820054$

Figure 5: Sums from $n=1$ up to shifted maximum shM84 for $n^{-1/2}\sin(R, \ln(n))$ function.

In conclusion, this study offers a rational explanation in this study for unique position of the Riemann critical line for all non-trivial zeros, what is one from seven well-known complex mathematical problems called Millennium Problem.

Limitations of this study and directions for future work

The values of R -constants in Euler–Riemann zeta function represent number distributions, suggesting a balance of chaotic and harmonic behaviors in a convergent function along both sin and cos functions at once. The observed mirror effect is an open question for the next.

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