Peertechz

Annals of Mathematics and Physics
MATHEMATICS AND PHYSICS GROUP
submitpaper@peertechz.com

ANNALS OF Mathematics and Physics OPEN ACCESS JOURNAL

ISSN: 2689-7636 DOI: https://dx.doi.org/10.17352/amp

Research Article

Heuristic Approach to Quantum Gravity in Terms of Incremental Curvature of Spacetime

James R Powell*

UT San Antonio Department of Physics and Astronomy, One UTSA Circle, San Antonio, TX 78249, USA

Received: 24 February, 2025
Accepted: 07 March, 2025
Published: 08 March, 2025

***Corresponding author:** James R Powell, PhD., UT San Antonio Department of Physics and Astronomy, One UTSA Circle, San Antonio, TX 78249, USA, E-mail: james.powell@utsa.edu

ORCiD: https://orcid.org/0000-0002-9247-7302

Keywords: Quantum gravity; Planck scale; Curvature entanglement; Compton wavelength

https://www.mathematicsgroup.us

Abstract

Physicists since Einstein have pondered how gravitation and quantum mechanics could be connected fundamentally given widely differing scales and mathematical formalism. Gravitational bending of light in null geodesics around an isolated point mass, curved incrementally to Compton's wavelength is proposed to associate semi-classical particle interactions with quantum wave phenomena. Although a basic methodology rather than a *full quantum* theory of gravity, this path leads to a simple expression of *gravity at the quantum level* in terms of constants ħ, c, and G, with mass as the source of gravitational curvature. When taken further to the Planck scale, *quantized* spacetime curvature is the square of the inverse Planck length which is a constant and the canonical inner boundary of spacetime. This elementary thought experiment of spacetime curvature taken to the Planck limit arrives at the ultimate state where quantum gravity existed in the very early universe. This state may also exist at the smallest scales in the current universe as *curvature entanglement* at nearly undetectable low energy due to the relative strength of gravity.

Introduction

A theory of Quantum Gravity (QG) has remained elusive for 100 years since Einstein first wondered how the new theory of quantum mechanics could possibly combine with gravitation in a unified field theory. This paper proposes a rudimentary approach to QG from first principles independent of currently presumed models or mathematical formalism, which might be verifiable experimentally as gravitational action at the quantum level. It is axiomatic that this investigative path should lead to the Planck scale as the realm of quantum gravity, and connect seamlessly with macro-gravitational theory by Bohr's correspondence principle so that quantum calculations somehow link with classical [1]. A simple thought experiment based on the gravitational bending of light as a variation of Compton's scattering experiment takes the gravitational curvature of spacetime to the quantum level. This natural progression to the Compton wavelength, then further extrapolated to the Planck scale, shows how this may be a form of quantum gravity. Combinations of ħ, c, and G are quantum-gravitational in terms of Planck units l_p, m_p, and t_p, altogether postulated as the natural domain of quantum gravity [2] (Figure 1).

The Planck units of mass, length, and time mark the scale where quantum effects of the gravitational interaction are expected to become important. Smolin's commonsense 'naive realism' would offer that we are limited in what we can know about extreme conditions at the beginning of the universe in the standard model of cosmology, or at the center of black holes, which are both beyond observation and experiment suggesting more intuitive approaches [3]. The very early universe (VEU) is such an extreme state at Planck energy ~ 10^{19} GeV at an initial Planck redshift, z ~ 10^{32} according to the standard model of cosmology. Distant galaxies can only be observed out to z ~ $10^{1.045}$ = 11.1 [4]. Similarly, the Large Hadron Collider (LHC) can generate proton collision energies approaching 14 TeV, or about 13 orders of magnitude below the Planck energy at length scales well above the Planck length, Hossenfelder [5]. "The scale where quantum gravity is necessary to describe space and

Citation: Powell JR. Heuristic Approach to Quantum Gravity in Terms of Incremental Curvature of Spacetime. Ann Math Phys. 2025;8(2):029-034. Available from: https://dx.doi.org/10.17352/amp.000143

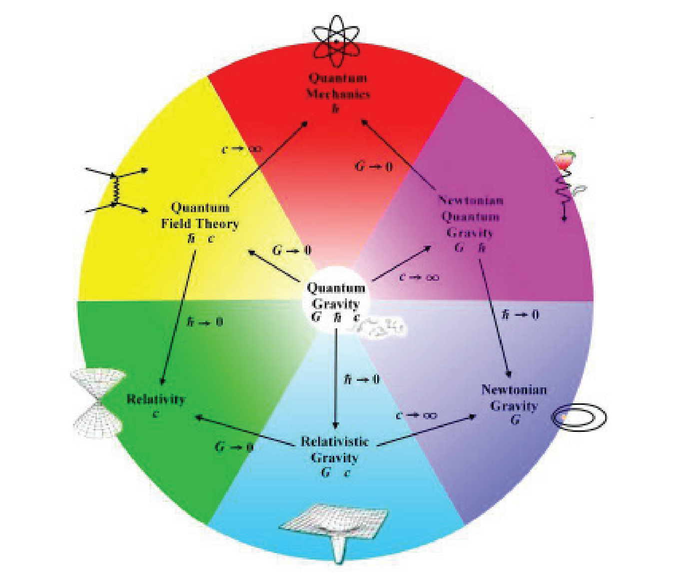

Figure 1: Quantum Gravity in terms of ħ, c, and G at the Planck Scale (universe-review.ca).

time is the Planck scale" [6]. The proposed heuristic thought experiment is a straightforward alternative to theoretical extrapolations and models with ever-increasing layers of mathematical formalism which can lose meaning and are unverifiable. We begin with a spherically symmetric geometry around a central point mass to isolate gravitational action in terms of the curvature of spacetime, which is then taken to the quantum scale to understand the quantum-gravitational connection at a fundamental level. Compton states in his 1923 paper, "This remarkable agreement between our formulas and the experiments can leave but little doubt that the scattering of X-rays is a quantum phenomenon" [7]. This means that the deflection of light due to curvature of spacetime as scattering around a central mass is also a quantum phenomenon. Quantum-gravitational curvature as the physically meaningful observable desired in [3] is also being addressed by other authors, see e.g. [8]. This heuristic approach to *gravity at the quantum level* as a form of quantum gravity is one of Smolin's "Three roads to quantum gravity" on "the road from relativity to include quantum phenomena" [6].

Quantum-level gravitational curvature of spacetime

Compton's scattering experiments with X-rays in the early days of quantum mechanics combined classical collisions of particles with wave behavior to explain observed wavelength shifts using his famous equation [7]. Compton's wavelength as the quantum property of a particle may also be used to explore quantum wave behavior in terms of gravitational curvature in the region of a point mass, Figure 2, as a heuristic thought experiment that is also scattering in a broader sense. Gravitational curvature of spacetime around an isolated, non-spinning point mass can be simply written as

$$K(r) = -Gm/(c^2r^3) \quad (1)$$

(4.4.1) [9], where K(r) is spacetime curvature at a distance r from the point mass m, with G and c constants of gravitation and the speed of light. The minus sign denotes inward curvature. Berry states that this equation emerges rigorously from the spherically symmetric Schwarzschild spacetime solution to Einstein's general relativity equations [9,10] as shown in the Appendix. Equation (1) is similar in form and simplicity to gravitational lensing as angle, $\alpha = 4GM/(c^2b)$ [11-13] as the impact factor b, or distance from the central point mass, *m*. Our thought experiment begins with Compton's laboratory configuration, but with an offset photon path, and no particle 'collision' in the classical sense, just 'curvature scattering'. We can then shift the photon path closer to central mass *m* in incremental steps. This is accomplished by taking equation (1) to the quantum level by scaling r from large-scale General Relativity to the quantum Compton wavelength, $\lambda_c = h/(mc)$ [7] with *h* the non-reduced Planck's constant. Then K(r) at the limit of the Compton wavelength, quantum mechanical by definition,

$$K(\lambda_c) = -Gcm^4/h^3 = -Gcm^4/(8\pi^3\hbar^3) \quad (2)$$

This is gravitational spacetime curvature at the Compton length from a point particle of mass m in terms of the constants G, c, and h or ħ and mass *m* with the constraint of spherically symmetric geometry, just as with the Schwarzschild solution to Einstein's equations, and given λ_c >> the characteristic size of m. The right-hand equation is in terms of the reduced Planck's constant. The Compton wavelength is thus a connection factor between quantum radiation and classical particle interactions as derived by Compton in his x-ray scattering experiments. Equation (2) has all the constants expected of a quantum gravity equation, and although a simple expression, it quantifies gravitational curvature at the necessary quantum-Compton wavelength scale. Since the central mass and photon are interacting gravitationally in terms of curvature $K(\lambda_c)$, (2) could be considered 'gravitational entanglement', and is unifying in the sense it incorporates constants of gravitation, relativity, and quantum mechanics which are also the constants of the Planck scale as well as quantum gravity. Mass can also be considered a constant, and thus (2) is constant for a given central point particle, and there are arguments suggesting that mass can be considered an operator in quantum mechanics [14-16]. An operator **m** as a component of energy and momentum operators would make (2) a fully quantum expression, or at least semiclassical as a hybrid representation of quantum-gravitational curvature. Spacetime curvature surrounding a central mass is the same effect as the gravitational deflection of light/gravitational lensing first proposed by Einstein [11] as mentioned above and shown below in Figure 2, but in terms of deflection angle in radians rather than K(r) in m^{-2}. To compare with (2), this deflection angle is

$$\alpha(b) = 4GM/(c^2b) \quad (3)$$

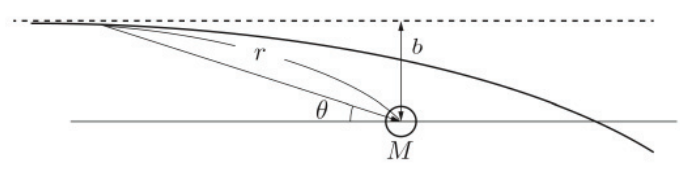

Figure 2: Trajectory of a Light Ray.

Citation: Powell JR. Heuristic Approach to Quantum Gravity in Terms of Incremental Curvature of Spacetime. Ann Math Phys. 2025;8(2):029-034. Available from: https://dx.doi.org/10.17352/amp.000143

where b is again the impact factor or radial distance from the central mass M, or as the center of an extended spherical mass, but with characteristic size $r_m < b$ as with an ideal point particle. Any 'wobble' of M due to the slight momentum effect of the traversing photon is disregarded in the reference frame of M.

Scattering here is thus a more general definition of deflection of light without displacing the central particle as in Compton's experiment. This is the same effect as gravitational redshift or decreasing photon energy with a corresponding increase in wavelength. Gravitational redshift is defined as $z \equiv (\lambda_o - \lambda_e)/\lambda_o = (\nu_e - \nu_o)/\nu_e = GM/(c^2r)$ (2.3.4) [9] as an energy loss caused by the gravitational field effect on the photon, and although dimensionless, the right-hand side here is similar in form to (1) and Compton's wavelength equation in [7].

Quantum gravitational curvature at the planck scale of quantum gravity

Compton gravitational curvature in equation (2) expresses spacetime curvature at the quantum level with all the constants expected for quantum gravity just like Planck units for mass, length, and time as unique combinations of ħ, c, and G, with mass the source of curvature at $r = \lambda_C$. Using the experimental h as used by Planck in his initial proposal of universal units [17], as an example we take the central mass to be an electron which Compton used in the derivation of his scattering equation and wavelength,

$$K(\lambda_{Ce}) = -Gcm_e^4/h^3 = -4.74 \times 10^{-23}\ m^{-2}, \quad (4)$$

which is the gravitational curvature of a geodesic path at the Compton length from the electron. Since it has long been established that quantum gravity exists at the Planck scale where gravity and quantum effects necessarily combine, the Planck scale curvature of equation (2) is

$$K(\lambda_{CP}) = -Gcm_p^4/\hbar^3 = -c^3/(\hbar G) = -l_p^{-2} \approx 3.91 \times 10^{69}\ m^{-2}, \quad (5)$$

here with the reduced Planck's constant and disregarding the factor of 8π. This is an important result that validates equation (2) to the extreme curvature of the Planck scale far from what is measurable in the current large-scale universe. If we apply the Planck mass and length directly in equation (1) [9],

$$K(l_p, m_p) = -Gm_p/(c^2l_p^3) = -c^3/(\hbar G) = -l_p^{-2}, \quad (6)$$

we arrive at the same result as in (5) as the geodesic path is reduced to the Planck core of spacetime, and again a constant which means there can be no further curvature increase beyond this limit. This is consistent with Birrell and Davies who state that the square of the inverse Planck length appears in the role of *coupling constant* in K(r) in units of m^{-2} which may mark the threshold of a full theory of quantum gravity [18]. This demonstrates how the heuristic path leads to the Planck scale of quantum gravity, and points to an ultimate limit which may be the boundary or foundation of spacetime. The connection to the early universe of the Planck scale is significant since the spherically symmetric Schwarzschild solution is ideally suited to this era. The Planck length here connects large-scale gravitation to the limit of quantum theory ($\Delta E \Delta t \sim \hbar \rightarrow E_p t_p = \hbar$ at the Planck scale near the beginning), only combined in the domain where quantum gravity once existed in the early universe at the extreme limits of the Planck scale.

We note that the form of (5) and (6) is $K(l) = 1/l^2$, length l as a radius which is the same as for a sphere with $K \equiv 1/R^2$ (4.3.2) [9], R the radius of a sphere, except that (2) and (4)-(6) are negative as inward curvature contrasted with positive curvature for the sphere. Berry clarifies that for spherical geometry curvature may be negative which means that the circumference C exceeds that of a circle of 2πa, and the negative-curvature surface would be inward-curving or somewhat 'saddle-shaped' when embedded in a three-dimensional space. Accordingly, this negative inward curvature 'saddle-shaped sphere' of radius l_p has extreme curvature of magnitude $1/l_p^2 \sim 1/10^{70}\ m^{-2}$. This assumes that shape or even particles themselves other than photons retain any meaning at the Planck scale.

QG spacetime curvature calculations

We can test equation (2) at energies well below the Planck scale with more approachable calculations. If we choose the central particle to be an electron, we can approximate Compton's scattering experiment with the high-energy photon path offset at r. The Compton wavelength for an electron is

$$\lambda_{Ce-} = h/(m_ec) = 2.42 \times 10^{-12}\ m \quad (7)$$

which is much greater than the radius of an electron, $r_e \sim 10^{-15}$ m, so within the constraint that r is much greater than the characteristic radius of the particle. For the gravitational curvature in the case of the electron in (4),

$$K(\lambda_{Ce}) = -Gcm_e^4/h^3 = 4.74 \times 10^{-24}\ m^{-2} \quad (8)$$

which is the curvature of spacetime for a photon passing by an electron at the Compton radius as shown earlier. For an order of magnitude comparison with spacetime curvature around the sun as first determined by Eddington on May 29, 1919 [19], we can replicate his calculation using Berry's equation (1),

$$K(R_\odot) = -GM_\odot/(c^2R_\odot^3) = -4.38 \times 10^{-24}\ m^{-2}, \quad (9)$$

which is close to (8). Values in (8) and (9) would be detectable by current experimental apparatus much advanced beyond Eddington's possibly with gravitational wave detectors or other means. His observation was made during a total eclipse of the sun to test Einstein's prediction from general relativity that light should travel in curved geodesic paths in spacetime warped by an object's gravitational field, here just skirting the edge of the sun or bending 1.75" as observed by Eddington. Although an example calculation, it demonstrates that the more testable conditions of QG curvature of equations (4) and (8) are within the scope of observation as with (9), providing an empirical framework for our thought experiment before extrapolating to the Planck scale which is not currently observable or testable with existing equipment.

To extend QG curvature to black holes, Hawking radiation is quantum gravitational as derived from quantum field theory as an extension of our thought experiment. Spacetime curvature at the Schwarzschild radius of a black hole is $K(R_S)$ where R_S

Citation: Powell JR. Heuristic Approach to Quantum Gravity in Terms of Incremental Curvature of Spacetime. Ann Math Phys. 2025;8(2):029-034. Available from: https://dx.doi.org/10.17352/amp.000143

$= 2GM/c^2$. Hawking radiation has a thermal spectrum at T_{BH} as the Bekenstein-Hawking temperature [2,20] emitted at the Schwarzschild radius with spacetime curvature $K(R_S)$. If we take equation (9) to R_S, then

$$K(R_S) = -GM/(c^2R_S^3) = -c^4/(8G^2M^2) = -(1/2)(1/R_S^2) \quad (10)$$

where M is the mass of the black hole. This is consistent with our earlier analysis quantifying curvature of spacetime as $1/R^2$ in (5) and (6) and just before Section 2. As above, we know from Hawking [20] and Wald [21] that a black hole radiates a thermal spectrum at Bekenstein-Hawking temperature, Pathria and Beale [22],

$$T_{BH} = \hbar\kappa/(2\pi ck_B) = \hbar c^3/(8\pi k_B GM), \quad (11)$$

where surface gravity $\kappa = c^4/(4GM)$ (14.3.8) [21]. Wald states that the temperature in (11) is precisely that for a perfect blackbody emitter. Since a blackbody spectrum requires the discrete constant *h* in Planck's equation to match the observed thermal spectrum, and this radiation arises from surface gravity (κ), then (11) is a quantum gravity equation and general expression beyond the black hole case. We can rewrite (11) in terms of both classical/statistical thermal radiation and quantum energy, and separating terms,

$$k_BT_{BH} = \hbar c^3/(8\pi GM) = (\hbar c/G)(c^2/8\pi M) = m_P^2c^2/(8\pi M), \quad (12)$$

with $(\hbar c/G)$ the m_P^2 connection to the Planck scale. The left side is classical thermal radiation energy k_BT at the equipartition value, and the right side is quantum gravitational energy at the Planck scale. If we now let the black hole mass M be equal to m_P, then

$$k_BT_{BH} = m_Pc^2/8\pi \approx (\hbar c^5/G)^{1/2} = E_P, \quad (13)$$

the Planck energy at ~ 10^{19} GeV. Primordial black holes (PBHs) postulated by Hawking in the very early universe [23] with Planck mass as the limit, would emit a blackbody spectrum at Planck temperature (Planck mini-black holes that possibly existed in the primordial universe would have evaporated by now). So, from the Bekenstein/Hawking equation for black hole temperature, necessarily quantum gravitational, we again arrive at the Planck scale of quantum gravity at extreme Planck energy and temperature. This is consistent with the Bekenstein/Hawking temperature of a PBH of Planck mass near the Planck scale, $T_{BH}(m_P) = T_P/8\pi$. Detecting Hawking radiation is an interesting variation of the thought experiment since Hawking photon escape from a black hole is gravitational, thermodynamic, and quantum in nature, and therefore Bekenstein-Hawking temperature is quantum-gravitational [2]. Equation (13) is semi-classical with temperature being a classical statistical parameter, but depends on Planck's constant in Hawking's temperature equation.

Finally, Jacob Bekenstein wrote his universal entropy bound as

$$S/E \le 2\pi k_BR/(\hbar c) \quad (14)$$

[24], here including Boltzmann's constant for dimensional consistency; and where S and E are the entropy and energy in a spherical volume of radius R. If we invert this expression as an equality with ξ some unknown entropy factor (possibly with an initial range $1/(2\pi) \le \xi \le 2\pi$, the left side of the inequality at the Planck scale, and right side for current large-scale entropy). Then,

$$E/S = \hbar c/(2\pi\xi k_BR) \quad (15)$$

where this is now a ratio of energy to entropy in a spherical volume written as an equality. If we then let $R \to R_S = 2GM/c^2$ as a characteristic gravitational radius associated with any mass M from Schwarzschild's solution to Einstein's field equations, then

$$E/S = \hbar c^3/(8\pi k_BGM) = T_{E/S} = T_{BH}, \quad (16)$$

setting $\xi = 2$ near the upper limit of ξ in (15), and showing that equation (16) is also the Bekenstein-Hawking temperature of a black hole. Equation (15) is, therefore, a more general expression describing the thermodynamic energy-to-entropy ratio in a system, or the margin of energy order over disorder, rather than merely the temperature of a Schwarzschild black hole. If (16) is a more general expression of temperature as a measure of 'energy order' from Bekenstein's entropy bound, then it may extend to the Planck scale of quantum gravity. If we let the only variable M be equal to the Planck mass, then $E/S = T_P$, the Planck temperature (neglecting factor 8π). This shows the consistency of (15) to the Planck scale as the scale of quantum gravity, and equations (15) and (16) can reasonably be considered equations of thermal quantum gravity with all the appropriate constants. An experiment has been proposed by the University of Maryland-NIST [25] to test *gravitational entanglement* with the superposition of states at two locations in the gravitational field using a double-slit interferometer combined with a single atom pendulum. This is intended to show how a massive particle can indeed be entangled by gravity as shown by its interference pattern. An expectation is that the intensity of this QG interference pattern will be proportional to gravitational spacetime curvature and carried by gravitons as the exchange particle (if they exist) like photons in the electromagnetic field. This experiment is planned for the sub-atomic level near the Compton wavelength where we derived the heuristic path above, and where it seems necessary to pursue quantum gravity. Another approach to QG at energies below the Planck scale is proposed in Wallace's recent "Quantum Gravity at Low Energies" (LEQG) [26]. This paper parallels the above discussion as a synthesis of ideas such as effective field theory, with constraints of 'nonrenormalizability', but with inevitable breakdown at the Planck scale. We have associated the curvature of spacetime, here at the Schwarzschild radius, to quantum-level curvature at the Compton length in equations (2) and (4), with the inverse of Bekenstein's entropy bound an indicator of energy order in (15) and (16), and how these converge quantum-gravitationally to the Planck scale.

Conclusion

We know from general relativity that the curvature of spacetime, K(r) is a measure of gravitational strength at radius r. As *r* is reduced to the Compton wavelength we are

Citation: Powell JR. Heuristic Approach to Quantum Gravity in Terms of Incremental Curvature of Spacetime. Ann Math Phys. 2025;8(2):029-034. Available from: https://dx.doi.org/10.17352/amp.000143

necessarily in the quantum realm. $K(\lambda_C)$ in equation (2) is a quantized form of gravitational curvature at the Compton wavelength dependent on the only variable, mass as the source of curvature. In this sense, a quantum of gravitational curvature is quantum gravity. Although not incorporating all associated quantum characteristics desired in a full theory of quantum gravity, this is a baseline approach to QG in a simple form as '*gravitationally induced curvature entanglement*'. It is possible that fully unified quantized gravity may have existed only in the VEU when it was appropriately small at high energy at the Planck scale. It is natural then to take Equation (2) to the Planck scale in the domain of quantum gravity which results in $K(l_P) = -1/l_P^2$ at the presumed Planck horizon limit of spacetime, which is a constant. Equations (2) and (4) - (6), as quantum-gravitational interactions include the expected constants c, ħ, and G of quantum gravity. The simple intuitive arguments and equations proposed here show how macro and micro scales were once merged in the VEU, but a merger which now either no longer exists, or is undetectable in the current large-scale universe requiring subatomic-level approaches such as in [25] and LEQG in [26]. A variation of Compton's laboratory result is a proposed thought experiment and analytical path to a fundamental expression of gravitational curvature at the quantum level in (2) with the objective of providing an analytic basis for quantum-gravitational curvature of spacetime as canonical quantum gravity. Smolin references Leibniz' "Principle of Sufficient Reason" (PSR) which applied here would mean that a complete understanding of gravity would be as curvature of spacetime [3]. Quantum gravity could then be gravitational curvature at the quantum level valid to the Planck scale. Rovelli states "Quantum gravity is the name given to any theory that describes gravity in the regimes where quantum effects cannot be disregarded." [27]. Equation (2) certainly applies. Rovelli continues, "Simple dimensional arguments show that the physical phenomenon where quantum gravitational effects become relevant are those characterized by the length scale $l_{Planck} = (\hbar G/c^3)^{1/2} \sim 10^{-33}$ cm, called the Planck length." This is confirmed by equation (5). Freeman Dyson has said we live in the classical world of gravitation which is deterministic and involves measurements that are historical; and the quantum world is inherently future and probabilistic, not directly accessible until observed and measured, which then becomes classical. He also said that the two worlds do not appear to be (fully) unified as observed, and why do they have to be [28]. There is no physical principle requiring quantum mechanics and general relativity to be combined into one all-encompassing theory other than supposed for a grand unified theory of all interactions, and merging GR and QM may not include the full mathematical formalism of either. Gravity and quantum mechanics have remained distinct theories with unique mathematical descriptions separated by scale and the probabilistic nature of quantum mechanics, and historical time and scale of relativity since the epoch of the very early universe. It remains to be seen how Einstein's general relativity equations and the energy-momentum tensor could be conformed to a purely quantum mechanical description with the mathematical formalism of the wave equation, superposition of states, entanglement, and Heisenberg uncertainty. Gravitational entanglement in terms of incremental, quantum-level spacetime curvature proposed here is offered as a fresh, provisional approach.

"After all, atoms do fall, so the relationship between gravity and the quantum is not a problem for nature." Lee Smolin [6].

Acknowledgement

Special thanks to Professor Michael Berry for his insight and simple equation for gravitational curvature of spacetime, and how this equation is derived from the Schwarzschild solution to Einstein's field equations. And to Freeman Dyson who was unafraid to challenge quantum gravity orthodoxy.

(Appendix)

References

1. Bohr N. About the Serial Spectra of the Elements. Zeitschrift für Physik. 1920;2(5):423-469. Available from: https://www.scirp.org/reference/referencespapers?referenceid=3254592
2. Lambourne R. Relativity, Gravitation and Cosmology. Cambridge University Press. 2010. Available from: https://www.cambridge.org/in/universitypress/subjects/physics/cosmology-relativity-and-gravitation/relativity-gravitation-and-cosmology?format=PB&isbn=9780521131384
3. Smolin L. Einstein's Unfinished Revolution. New York: Penguin Press. 2019. Available from: https://www.amazon.in/Einsteins-Unfinished-Revolution-Lee-Smolin/dp/014197916X
4. Oesch P, Brammer G, van Dokkum PG, Illingworth GD, Bouwens RJ, Labbé I, et al. A Remarkably Luminous Galaxy at Z=11.1 Measured with Hubble Space Telescope GRISM Spectroscopy. arXiv:1603.00461v1 [astro-ph.GA]. 2016 Mar 1. Available from: https://ui.adsabs.harvard.edu/abs/2016ApJ...819..129O/abstract
5. Hossenfelder S. Minimal Length Scale Scenarios for Quantum Gravity. Living Rev Relativ. 2013;16(2). Available from: https://link.springer.com/article/10.12942/lrr-2013-2
6. Smolin L. Three Roads to Quantum Gravity. New York: Basic Books/Perseus Books Group. 2001. Available from: https://www.amazon.in/Three-Roads-Quantum-Gravity-Smolin/dp/0465094546
7. Compton A. A Quantum Theory of the Scattering of X-Rays by Light Elements. Phys Rev. 1923;21(5):483-502. Available from: https://journals.aps.org/pr/abstract/10.1103/PhysRev.21.483
8. Amelino-Camelia G, Rosati G, Bedic S. Phenomenology of Curvature-Induced Quantum Gravity Effects. arXiv:2012.07790v1 [gr-qc]. 2020. Available from: https://doi.org/10.48550/arXiv.2012.07790
9. Berry M. Principles of Cosmology and Gravitation. Cambridge, UK: Cambridge University Press. 1976. Available from: https://archive.org/details/principlesofcosm0000berr/page/n7/mode/2up
10. Personal Communication with Prof. Michael Berry. H H Wills Physics Laboratory, Bristol, UK. 2020. Available from: https://www.bristol.ac.uk/people/person/Michael-Berry-f4aba075-921c-4697-a1d3-32256fa63b02/
11. Tavoro M. Einstein's Gravity, The Bending of Light and How He Became the World's Most Famous Scientist, An Explanation of the Gravitational Bending of Light by the Sun. Available from: https://medium.com/towards-data-science/einsteins-gravity-theory-and-the-bending-of-light-by-the-sun-1e796626dc19
12. Alexandrov A, Sliuser V, Zhdanov V. Verification of Einstein's Formula for Gravitational Deflection of Light Using Observations of Galactic Microlensing.

Citation: Powell JR. Heuristic Approach to Quantum Gravity in Terms of Incremental Curvature of Spacetime. Ann Math Phys. 2025;8(2):029-034. Available from: https://dx.doi.org/10.17352/amp.000143

arXiv:2004.07522 [gr-qc]. 2020 Apr 16. Available from: https://doi.org/10.48550/arXiv.2004.07522

13. Bartelmann M, Maturi M. Weak Gravitational Lensing. arXiv:1612.06535v1 [astro-ph.CO]. 2016 Dec 20. Available from: https://doi.org/10.48550/arXiv.1612.06535
14. Wessel W. On Relativistic Quantum Mechanics and the Mass Operator. Phys Rev. 1951;83(5):1031-1037. Available from: https://journals.aps.org/pr/abstract/10.1103/PhysRev.83.1031
15. Greenberger D. Conceptual Problems Related to Time and Mass in Quantum Theory. (Paper prepared for a talk at a conference at Oxford University honoring Prof. John Polkinghorne on his 80th birthday, Sept. 2010). arXiv:1011.3709. Available from: https://doi.org/10.48550/arXiv.1011.3709
16. Landau L, Lifshitz E. The Classical Theory of Fields: Course of Theoretical Physics, Fourth Revised English Edition. Pergamon Press; 1975. Available from: https://www.aldebaran.cz/studium/books/1994_Landau_Lifshitz-Fields.pdf
17. Planck M. The Theory of Heat Radiation. New York: Dover Publications, Inc.; 1914. Available from: https://gutenberg.org/files/40030/40030-pdf.pdf
18. Birrell N, Davies P. Quantum Fields in Curved Space. Cambridge University Press; 1984. Available from: https://www.cambridge.org/core/books/quantum-fields-in-curved-space/95376B0CAD78EE767FCD6205F8327F4C
19. Eddington A. Report on the Relativity Theory of Gravitation. The Physical Society of London. 1920. Available from: https://archive.org/details/reportontherelat028829mbp
20. Hawking S. Particle Creation by Black Holes. Commun Math Phys. 1975;43:199-220. Available from: https://link.springer.com/article/10.1007/BF02345020
21. Wald R. General Relativity. Chicago and London: The University of Chicago Press. 1989. Available from: https://cdn.preterhuman.net/texts/science_and_technology/physics/General_Relativity_Theory/General%20Relativity%20-%20R.%20Wald.pdf
22. Pathria R, Beale P. Statistical Mechanics. 3rd ed. Elsevier. 2011. Available from: http://linux0.unsl.edu.ar/~froma/MecanicaEstadistica/Bibliografia/PathriaBeale.pdf
23. Hawking S. Gravitationally Collapsed Objects of Very Low Mass. Mon Not R Astr Soc. 1971;152:75-78. Available from: https://doi.org/10.1093/mnras/152.1.75
24. Bekenstein J. Universal Upper Bound on the Entropy-to-Energy Ratio for Bounded Systems. Phys Rev D. 1981;23:287-342. Available from: https://journals.aps.org/prd/abstract/10.1103/PhysRevD.23.287
25. Carney D, Müller H, Taylor J. Using an Atom Interferometer to Infer Gravitational Entanglement Generation. PRX Quantum. 2021;2:030330. Available from: https://journals.aps.org/prxquantum/abstract/10.1103/PRXQuantum.2.030330
26. Wallace D. Quantum Gravity at Low Energies. arXiv:2112.1223 [gr-qc]. 2021. Available from: https://arxiv.org/abs/2112.12235
27. Rovelli C. Quantum Gravity. Scholarpedia. 2008;3(5):7117. Available from: http://www.scholarpedia.org/article/Quantum_gravity
28. Discussion with Freeman Dyson, "Freeman Dyson: Why General Relativity and Quantum Mechanics Can't Be Unified". 2017. Available from: https://www.youtube.com/watch?v=pcicI_GJGcM

Citation: Powell JR. Heuristic Approach to Quantum Gravity in Terms of Incremental Curvature of Spacetime. Ann Math Phys. 2025;8(2):029-034. Available from: https://dx.doi.org/10.17352/amp.000143