Peertechz

Annals of Mathematics and Physics

MATHEMATICS AND PHYSICS GROUP

ANNALS OF Mathematics and Physics OPEN ACCESS JOURNAL

ISSN: 2689-7636 DOI: https://dx.doi.org/10.17352/amp

Research Article

Successive differentiation of some mathematical functions using hypergeometric mechanism

MI Qureshi, Tafaz ul Rahman Shah* and Shakir Hussain Malik

Department of Applied Sciences and Humanities, Faculty of Engineering and Technology, Jamia Millia Islamia (A Central University), New Delhi - 110025, India

Received: 03 January, 2023
Accepted: 07 December, 2023
Published: 08 December, 2023

***Corresponding author:** Tafaz ul Rahman Shah, Department of Applied Sciences and Humanities, Faculty of Engineering and Technology, Jamia Millia Islamia (A Central University), New Delhi - 110025, India, Tel: 7889682575; E-mail: tafazuldiv@gmail.com

ORCiD: https://orcid.org/0000-0001-8196-9806

Keywords: Generalized hypergeometric function; Pochhammer symbol

Abstract

In this article, we obtain successive differentiation of some composite mathematical functions: $(z)^{-\frac{1}{2}}\sin^{-1}\sqrt{(z)}+\sqrt{(1-z)}$; $(z)^{\frac{1}{2}}\sin^{-1}\sqrt{(z)}+\sqrt{(1-z)}$; $\frac{4}{z}\left[1-\sqrt{(1-z)}+\ell n\left(\frac{1+\sqrt{(1-z)}}{2}\right)\right]$; $\frac{4}{z^2}\left[2\sqrt{(1-z)}-2+z-2z\ell n\left(\frac{1+\sqrt{(1-z)}}{2}\right)\right]$ and $-\frac{4}{z}\ell n\left(\frac{1+\sqrt{(1-z)}}{2}\right)$, using a hypergeometric approach as the successive differentiation of these functions can not be performed by any other mathematical technique.

2020 MSC: 33C05, 33B10

1. Introduction and preliminaries

The ${}_pF_q$ $(p,q\in\mathbb{N}_0)$ is the generalized hypergeometric series defined by (see, e.g., [1-6]):

$$ {}_pF_q\left[\begin{matrix}\alpha_1,\ldots,\alpha_p;\\ \beta_1,\ldots,\beta_q;\end{matrix}\ z\right]=\sum_{n=0}^{\infty}\frac{(\alpha_1)_n\cdots(\alpha_p)_n}{(\beta_1)_n\cdots(\beta_q)_n}\frac{z^n}{n!} \tag{1.1}$$

$$={}_pF_q(\alpha_1,\ldots,\alpha_p;\beta_1,\ldots,\beta_q;z),$$

Being a natural generalization of the Gaussian hypergeometric series ${}_2F_1$, where $(\lambda)_\nu$ denotes the Pochhammer symbol (for $\lambda,\nu\in\mathbb{C}$) defined by

$$(\lambda)_\nu:=\frac{\Gamma(\lambda+\nu)}{\Gamma(\lambda)}\quad(\lambda,\nu+\lambda\in\mathbb{C}\setminus\mathbb{Z}_0^-)$$

$$\begin{cases}1 & (\nu=0;\lambda\in\mathbb{C}\setminus\mathbb{Z}_0^-),\\ \lambda(\lambda+1)\ldots(\lambda+n-1) & (\nu=n\in\mathbb{N};\lambda\in\mathbb{C}),\end{cases} \tag{1.2}$$

Here Γ is the familiar Gamma function (see, e.g., [5, Section 1.1]), and it is assumed that $(0)_0:=1$, an empty product as 1, and that the variable z, the numerator parameters α_1, ..., α_p and the denominator parameters β_1, ...,β_q take on complex values, provided that no zero appear in the denominator of (1.1), that is, that

$$(\beta_j\in\mathbb{C}\setminus\mathbb{Z}_0^-;\ j=1,\ldots,q).$$

Here and elsewhere, let $\mathbb{Z}$, $\mathbb{R}$ and $\mathbb{C}$ be respectively the sets of integers, real numbers, and complex numbers, and let

$$\mathbb{N}:=\{1,2,3\ldots\};\mathbb{N}_0:=\mathbb{N}\cup\{0\};\mathbb{Z}_0^-:=\mathbb{Z}^-\cup\{0\}=\{0,-1,-2,-3,\cdots\}.$$

For more details of ${}_pF_q$ including its convergence, its various special and limiting cases, and its further diverse generalizations, one may refer to [7,8]. Certain identities associated with the ${}_pF_q$ and its generalizations, which are necessary for this work, are brought to mind.

See ref. [4, p.71, Q.No.(18)]

$$\frac{\sin^{-1}(z)}{(z)} = {}_2F_1\left[\begin{matrix}\frac{1}{2},\frac{1}{2}; \\ \frac{3}{2};\end{matrix}\; z^2\right]; \quad |z|<1. \tag{1.3}$$

See ref. [6, p.44, Eq.(8)]

$$(1-z)^{-a} = {}_1F_0\left[\begin{matrix}a\;; \\ -;\end{matrix}\; z\right]; \quad a\in\mathbb{C} \text{ and } |z|<1. \tag{1.4}$$

See ref. [9, p.155, Eq.(2.1)]

$$-\frac{4}{z}\ell n\left(\frac{1+\sqrt{1-z}}{2}\right) = {}_3F_2\left[\begin{matrix}1,1,\frac{3}{2}; \\ 2,2;\end{matrix}\; z\right]; \quad |z|<1. \tag{1.5}$$

Whenever the generalized hypergeometric function ${}_pF_q$, including ${}_2F_1$, can be expressed in terms of Gamma functions through summation of its specified argument, which may include unit or $\frac{1}{2}$ argument, the outcome holds significant value from both theoretical and practical perspectives.

The generalized hypergeometric series has classical summation theorems, including those of Gauss, Gauss second, Kummer, and Bailey for the ${}_2F_1$ series, as well as Watson's, Dixon's, Whipple's, and Saalschütz's summation theorems for the ${}_3F_2$ series and others. These theorems have significant importance in both theory and application.

From 1992 to 1996, Lavoie et al. [10-12] published a series of works that generalized the aforementioned classical summation theorems for the ${}_3F_2$ series of Watson, Dixon, and Whipple. They also presented many special and limiting cases of their results, which have been further extended and generalized by Rakha-Rathie [13], Kim et al. [14], and more recently by Qureshi et al. [15]. These results have also been verified, using computer programs such as Mathematica.

The emergence of extensively generalized special functions, such as (1.1), has sparked intriguing research into their reducibility. Bhat et al. introduce certain hypergeometric functions involving arcsine (x) using the Maclaurin series and their applications [16]. Qureshi et al. [17] also introduced hypergeometric forms of some composite functions containing arccosine (x) using the same series. Many papers from Qureshi et al.[18,19] introduced hypergeometric forms of some functions involving arcsine (x) using a differential equation approach and some mathematical functions via the Maclaurin series.

Inspired by the aforementioned papers, especially [9] comparing the resulting ordinary differential equations with standard ordinary differential equations of Leibnitz and Gauss, obtained some new hypergeometric functions. Our objective is to introduce successive differentiation of some composite functions by using a hypergeometric approach. For that we mention the hypergeometric forms of some composite functions in section 2, with their proof in section 3, using the series rearrangement technique. Applications of these hypergeometric forms in successive differentiation (mentioned in section 4), are given in section 5.

2. Hypergeometric forms of some mathematical functions

When $|z|<1$, the following hypergeometric forms of mathematical functions hold true:

$$\frac{4}{z^2}\left[2\sqrt{(1-z)}-2+z-2z\ell n\left(\frac{1+\sqrt{(1-z)}}{2}\right)\right] = {}_3F_2\left[\begin{matrix}1,1,\frac{3}{2}; \\ 2,3;\end{matrix}\; z\right]. \tag{2.1}$$

$$(z)^{\frac{1}{2}}\sin^{-1}\sqrt{(z)}+\sqrt{(1-z)} = {}_2F_1\left[\begin{matrix}\frac{-1}{2},\frac{-1}{2}; \\ \frac{1}{2};\end{matrix}\; z\right]. \tag{2.2}$$

$$\frac{4}{z}\left[1-\sqrt{(1-z)}+\ell n\left(\frac{1+\sqrt{(1-z)}}{2}\right)\right] = {}_3F_2\left[\begin{matrix}1,1,\frac{1}{2}; \\ 2,2;\end{matrix}\; z\right]. \tag{2.3}$$

$$(z)^{-\frac{1}{2}}\sin^{-1}\sqrt{(z)}+\sqrt{(1-z)} = 2.{}_2F_1\left[\begin{matrix}\frac{-1}{2},\frac{1}{2}; \\ \frac{3}{2};\end{matrix}\; z\right]. \tag{2.4}$$

3. Derivation of hypergeometric forms

In this section, using the series rearrangement technique, we derive the hypergeometric forms of some mathematical functions mentioned in section 2.

Proof of hypergeometric form (2.1)

$$\text{Let} \quad H_1(z) = \frac{4}{z^2}\left[2\sqrt{(1-z)}-2+z-2z\ell n\left(\frac{1+\sqrt{(1-z)}}{2}\right)\right]$$

$$= \frac{8(1-z)^{\frac{1}{2}}}{z^2}-\frac{8}{z^2}+\frac{4}{z}-\frac{8}{z}\ell n\left(\frac{1+\sqrt{(1-z)}}{2}\right).$$

Using the equations (1.4) and (1.5), we have

$$H_1(z) = \frac{8}{z^2}\,{}_1F_0\left[\begin{matrix}\frac{-1}{2}; \\ -;\end{matrix}\; z\right]-\frac{8}{z^2}+\frac{4}{z}+2\,{}_3F_2\left[\begin{matrix}1,1,\frac{3}{2}; \\ 2,2;\end{matrix}\; z\right]$$

Citation: Qureshi MI, ul Rahman Shah T, Malik SH (2023) Successive differentiation of some mathematical functions using hypergeometric mechanism. Ann Math Phys 6(2): 182-186. DOI: https://dx.doi.org/10.17352/amp.000100

$$=\frac{8}{z^2}\sum_{n=0}^{\infty}\frac{(\frac{-1}{2})_n z^n}{n!}-\frac{8}{z^2}+\frac{4}{z}+2\,{}_3F_2\left[\begin{matrix}1,1,\frac{3}{2};\\ \\ 2,2;\end{matrix}\quad z\right]$$

$$=\frac{8}{z^2}\left[1-\frac{z}{2}+\sum_{n=2}^{\infty}\frac{(\frac{-1}{2})_n z^n}{n!}\right]-\frac{8}{z^2}+\frac{4}{z}+2\,{}_3F_2\left[\begin{matrix}1,1,\frac{3}{2};\\ \\ 2,2;\end{matrix}\quad z\right]$$

$$=\frac{8}{z^2}\sum_{n=2}^{\infty}\frac{(\frac{-1}{2})_n z^n}{n!}+2\,{}_3F_2\left[\begin{matrix}1,1,\frac{3}{2};\\ \\ 2,2;\end{matrix}\quad z\right]. \tag{3.1}$$

Replacing n by $(n+2)$ in equation (3.1), we get

$$H_1(z)=\frac{8}{z^2}\sum_{n=0}^{\infty}\frac{(\frac{-1}{2})_{n+2} z^{n+2}}{(n+2)!}+2\,{}_3F_2\left[\begin{matrix}1,1,\frac{3}{2};\\ \\ 2,2;\end{matrix}\quad z\right]$$

$$=\frac{8}{z^2}\frac{(\frac{-1}{2})_2 z^2}{(1)_2}\sum_{n=0}^{\infty}\frac{(\frac{3}{2})_n z^n}{(3)_n}+2\,{}_3F_2\left[\begin{matrix}1,1,\frac{3}{2};\\ \\ 2,2;\end{matrix}\quad z\right]$$

$$=4\left(\frac{-1}{2}\right)\left(\frac{1}{2}\right)\sum_{n=0}^{\infty}\frac{(\frac{3}{2})_n z^n(1)_n}{(3)_n n!}+2\,{}_3F_2\left[\begin{matrix}1,1,\frac{3}{2};\\ \\ 2,2;\end{matrix}\quad z\right]$$

$$=-\sum_{n=0}^{\infty}\frac{(1)_n(\frac{3}{2})_n z^n}{(3)_n n!}+2\sum_{n=0}^{\infty}\frac{(1)_n(1)_n(\frac{3}{2})_n z^n}{(2)_n(2)_n n!}$$

$$=2\sum_{n=0}^{\infty}\frac{(1)_n(1)_n(\frac{3}{2})_n z^n}{(2)_n(2)_n n!}-\sum_{n=0}^{\infty}\frac{(1)_n(\frac{3}{2})_n z^n}{(3)_n n!}$$

$$=\sum_{n=0}^{\infty}\frac{(1)_n(\frac{3}{2})_n z^n}{n!}\left[\frac{2(1)_n}{(2)_n(2)_n}-\frac{1}{(3)_n}\right]$$

$$=\sum_{n=0}^{\infty}\frac{(1)_n(\frac{3}{2})_n z^n}{n!}\left[\frac{2\Gamma(1+n)}{\Gamma(2+n)\Gamma(2+n)}-\frac{\Gamma(3)}{\Gamma(3+n)}\right]$$

$$=\sum_{n=0}^{\infty}\frac{(1)_n(\frac{3}{2})_n z^n}{n!}\left[\frac{2\Gamma(1+n)}{(1+n)\Gamma(1+n)\Gamma(2+n)}-\frac{2}{(2+n)\Gamma(2+n)}\right]$$

$$=\sum_{n=0}^{\infty}\frac{(1)_n(\frac{3}{2})_n z^n}{n!}\left[\frac{2}{\Gamma(2+n)}\left(\frac{1}{(1+n)}-\frac{1}{(2+n)}\right)\right]$$

$$=\sum_{n=0}^{\infty}\frac{(1)_n(\frac{3}{2})_n z^n}{n!}\left[\frac{2}{(2)_n\Gamma(2)}\left(\frac{1}{(1+n)}-\frac{1}{(2+n)}\right)\right]$$

$$H_1(z)=\sum_{n=0}^{\infty}\frac{2(1)_n(\frac{3}{2})_n z^n}{(2)_n\Gamma(2)n!}\left[\frac{1}{(1+n)}-\frac{1}{(2+n)}\right].$$

After further simplification, we get the result (2.1).

Proof of hypergeometric form (2.2)

$$\text{Let } H_2(z)={}_2F_1\left[\begin{matrix}\frac{-1}{2},\frac{-1}{2};\\ \\ \frac{1}{2};\end{matrix}\quad z\right]-\sqrt{(1-z)}$$

$$={}_2F_1\left[\begin{matrix}\frac{-1}{2},\frac{-1}{2};\\ \\ \frac{1}{2};\end{matrix}\quad z\right]-(1-z)^{\frac{1}{2}}.$$

Using the equation (1.4), we have

$$H_2(z)=\sum_{r=0}^{\infty}\frac{(-\frac{1}{2})_r(-\frac{1}{2})_r z^r}{(\frac{1}{2})_r\, r!}-{}_1F_0\left[\begin{matrix}\frac{-1}{2};\\ \\ -;\end{matrix}\quad z\right]$$

$$=\sum_{r=0}^{\infty}\frac{(-\frac{1}{2})_r(-\frac{1}{2})_r z^r}{(\frac{1}{2})_r\, r!}-\sum_{r=0}^{\infty}\frac{(-\frac{1}{2})_r z^r}{r!}$$

$$=\sum_{r=0}^{\infty}\frac{(-\frac{1}{2})_r z^r}{r!}\left[\frac{(-\frac{1}{2})_r}{(\frac{1}{2})_r}-1\right]$$

$$=\sum_{r=0}^{\infty}\frac{(-\frac{1}{2})_r z^r}{r!}\left[\frac{\Gamma(-\frac{1}{2}+r)\Gamma(\frac{1}{2})}{\Gamma(-\frac{1}{2})\Gamma(\frac{1}{2}+r)}-1\right]$$

$$=\sum_{r=0}^{\infty}\frac{(-\frac{1}{2})_r z^r}{r!}\left[\frac{\Gamma(-\frac{1}{2}+r)\sqrt{\pi}}{(-2\sqrt{\pi})(-\frac{1}{2}+r)\Gamma(-\frac{1}{2}+r)}-1\right]$$

$$=\sum_{r=0}^{\infty}\frac{(-\frac{1}{2})_r z^r}{r!}\left[\frac{1}{1-2r}-1\right]$$

$$=\sum_{r=0}^{\infty}\frac{(-\frac{1}{2})_r z^r}{r!}\left[\frac{2r}{1-2r}\right]$$

$$H_2(z)=2\sum_{r=1}^{\infty}\frac{(-\frac{1}{2})_r z^r}{(r-1)!(1-2r)}. \tag{3.2}$$

Replacing r by $(r+1)$ in equation (3.2), we get

$$=2\sum_{r=0}^{\infty}\frac{(-\frac{1}{2})_{r+1} z^{r+1}}{(r+1-1)!(1-2r-2)}$$

Citation: Qureshi MI, ul Rahman Shah T, Malik SH (2023) Successive differentiation of some mathematical functions using hypergeometric mechanism. Ann Math Phys 6(2): 182-186. DOI: https://dx.doi.org/10.17352/amp.000100

$$H_2(z)=\frac{2(-\frac{1}{2})z}{(-1)}\sum_{r=0}^{\infty}\frac{(-\frac{1}{2}+1)_r\, z^r}{r!(1+2r)}.$$

After further simplification, we get the result (2.2).

Similarly, we can get the remaining hypergeometric forms (2.3) and (2.4) in the same way as the hypergeometric forms (2.1) and (2.2).

4. Successive differential coefficients of some mathematical functions

When $|z|<1$, successive differential coefficients of some mathematical functions hold true:

$$\frac{d^n}{dz^n}\left[-\frac{4}{z}\ell n\left(\frac{1+\sqrt{(1-z)}}{2}\right)\right]=\frac{(1)_n(1)_n(\frac{3}{2})_n}{(2)_n(2)_n}\,{}_3F_2\left[\begin{array}{c}1+n,1+n,\frac{3}{2}+n;\\ \\ 2+n,2+n;\end{array}\ z\right]. \tag{4.1}$$

$$\frac{d^n}{dz^n}\left[\frac{4}{z}\left\{1-\sqrt{(1-z)}+\ell n\left(\frac{1+\sqrt{(1-z)}}{2}\right)\right\}\right]=\frac{(1)_n(1)_n(\frac{1}{2})_n}{(2)_n(2)_n}\times$$
$$\times{}_3F_2\left[\begin{array}{c}1+n,1+n,\frac{1}{2}+n;\\ \\ 2+n,2+n;\end{array}\ z\right]. \tag{4.2}$$

$$\frac{d^n}{dz^n}\left[\frac{4}{z^2}\left\{2\sqrt{(1-z)}-2+z-2z\ell n\left(\frac{1+\sqrt{(1-z)}}{2}\right)\right\}\right]=\frac{(1)_n(1)_n(\frac{3}{2})_n}{(2)_n(3)_n}\times$$
$$\times{}_3F_2\left[\begin{array}{c}1+n,1+n,\frac{3}{2}+n;\\ \\ 2+n,\quad 3+n;\end{array}\ z\right]. \tag{4.3}$$

$$\frac{d^n}{dz^n}\left[\frac{\sin^{-1}\sqrt{(z)}}{\sqrt{(z)}}+\sqrt{(1-z)}\right]=\frac{2.(-\frac{1}{2})_n(\frac{1}{2})_n}{(\frac{3}{2})_n}\,{}_2F_1\left[\begin{array}{c}-\frac{1}{2}+n,\frac{1}{2}+n;\\ \\ \frac{3}{2}+n;\end{array}\ z\right]. \tag{4.4}$$

$$\frac{d^n}{dz^n}\left[\sqrt{(z)}\sin^{-1}\sqrt{(z)}+\sqrt{(1-z)}\right]=\frac{(-\frac{1}{2})_n(-\frac{1}{2})_n}{(\frac{1}{2})_n}\,{}_2F_1\left[\begin{array}{c}-\frac{1}{2}+n,-\frac{1}{2}+n;\\ \\ \frac{1}{2}+n;\end{array}\ z\right]. \tag{4.5}$$

5. Applications of hypergeometric forms in successive differentiation

In this section, using the series rearrangement technique, we give the proof of successive differential coefficients of mathematical functions mentioned in Section 4.

Proof of successive differential coefficient (4.1)

Let $D_1(z)=\frac{d^n}{dz^n}\left[-\frac{4}{z}\ell n\left(\frac{1+\sqrt{(1-z)}}{2}\right)\right].$

Using the equation (1.5), we have

$$D_1(z)=\frac{d^n}{dz^n}\left({}_3F_2\left[\begin{array}{c}1,1,\frac{3}{2};\\ \\ 2,2;\end{array}\ z\right]\right)$$

$$=\frac{d^n}{dz^n}\left(\sum_{r=0}^{\infty}\frac{(1)_r(1)_r(\frac{3}{2})_r\, z^r}{(2)_r(2)_r\, r!}\right)$$

$$=\frac{d^n}{dz^n}\left(\sum_{r=0}^{n-1}\frac{(1)_r(1)_r(\frac{3}{2})_r\, z^r}{(2)_r(2)_r\, r!}+\sum_{r=n}^{\infty}\frac{(1)_r(1)_r(\frac{3}{2})_r\, z^r}{(2)_r(2)_r\, r!}\right)$$

$$=0+\sum_{r=n}^{\infty}\frac{(1)_r(1)_r(\frac{3}{2})_r}{(2)_r(2)_r\, r!}\frac{d^n}{dz^n}z^r$$

$$D_1(z)=\sum_{r=n}^{\infty}\frac{(1)_r(1)_r(\frac{3}{2})_r\, z^{(r-n)}r!}{(2)_r(2)_r\,(r-n)!r!}. \tag{5.1}$$

Replacing r by $(r+n)$ in equation (5.1), we get

$$D_1(z)=\sum_{r=0}^{\infty}\frac{(1)_{r+n}(1)_{r+n}(\frac{3}{2})_{r+n}\, z^{(r+n-n)}}{(2)_{r+n}(2)_{r+n}\,(r-n+n)!}$$

$$=\frac{(1)_n(1)_n(\frac{3}{2})_n}{(2)_n(2)_n}\sum_{r=0}^{\infty}\frac{(1+n)_r(1+n)_r(\frac{3}{2}+n)_r\, z^r}{(2+n)_r(2+n)_r\, r!}.$$

After further simplification, we get the result (4.1).

Similarly, we can get the remaining successive differential coefficients (4.2)-(4.5) in the same way as the successive differential coefficient (4.1).

Conclusion

In this paper, we have obtained the hypergeometric forms of some composite functions. Further, we have found some applications of these hypergeometric forms in successive differentiation. We conclude our present investigation with the remark that the successive differentiation of some other functions can be derived in an analogous manner. Moreover, the results deduced above are quite significant and are expected to lead to some potential applications in several diverse fields of mathematical, physical, statistical, and engineering sciences.

Acknowledgment

The authors wish to extend their heartfelt appreciation to the anonymous reviewers for their invaluable feedback. Their constructive and encouraging comments have greatly contributed to enhancing the quality of this paper.

References

1. Andrews GE, Askey R, Roy R. Special Functions, Encyclopedia of Mathematics and its Applications.Cambridge University Press. 71:1999.

2. Bailey WN. Generalized Hypergeometric Series, Cambridge Math. Tract No. 32, Cambridge Univ. Press, Cambridge, 1935; Reprinted by Stechert-Hafner, New York, 1964.

3. Lebedev NN. Special Functions and their Applications, (Translated by Richard A. Silverman), Prentice-Hall, Inc., Englewood Cliffs, New Jersey. 1965.

4. Rainville ED. Special Functions, The Macmillan Co. Inc., New York 1960; Reprinted by Chelsea publ. Co., Bronx, New York 1971.

5. Srivastava HM, Choi J. Zeta and q-Zeta Functions and Associated Series Integrals, Elsevier Science Publishers, Amsterdam, London and New York, 2012.

6. Srivastava HM, Manocha HL. A Treatise on Generating Functions, Halsted Press (Ellis Horwood Limited, Chichester), John Wiley and Sons, New York, Chichester Brisbane, Toronto, 1984.

7. Exton H. Multiple Hypergeometric Functions and Applications, Halsted Press (Ellis Horwood, Chichester), John Wiley and Sons, New York, London, Sydney and Toronto, 1976.

8. Slater LJ. Generalized Hypergeometric Functions, Cambridge University Press, Cambridge, London and New York, 1966.

9. Qureshi MI, Malik SH, Ara J. Hypergeometric forms of some mathematical functions via differential equation approach. 2020; 50(2): 153-159.

10. Lavoie JL, Grondin F, Rathie AK. Generalizations of Watson's theorem on the sum of a ${}_3F_2$, Indian J Math. 1992; 34(2): 23-32.

11. Lavoie JL, Grondin F, Rathie AK, Arora k. Generalizations of Dixon's theorem on the sum of a ${}_3F_2$, Math Comput 1994; 62(205): 267–276. https://doi.org/10.2307/2153407

12. Lavoie JL, Grondin F, Rathie AK. Generalizations of Whipple's theorem on the sum of a ${}_3F_2$, J. Comput. Appl. Math. 1996; 72(2): 293–300. https://doi.org/10.1016/0377-0427(95)00279-0

13. Rakha MA, Rathie AK. Generalizations of classical summation theorems for the series ${}_2F_1$ and ${}_3F_2$ with applications, Integral Transform. Spec Funct. 2011; 22 (2011); 823-840. http://dx.doi.org/10.1080/10652469.2010.549487

14. Kim YS, Rakha MA, Rathie AK. Extensions of certain classical summation theorems for the series ${}_2F_1$, ${}_3F_2$ and ${}_4F_3$ with applications in Ramanujan's summations. Int J Math Math Sci. 2010; 26: 309503.

15. Qureshi MI, Choi J, Shah TR. Certain Generalizations of Quadratic Transformations of Hypergeometric and Generalized Hypergeometric Functions. Symmetry. 2022; 14: 1073. https://doi.org/10.3390/sym14051073.

16. Bhat AH, Qureshi MI, Majid J. Hypergeometric forms of certain composite functions involving arcsine (x) using Maclaurin series and their applications. 2020; 50(2): 139-145.

17. Qureshi MI, Majid J, Bhat AH. Hypergeometric forms of some composite functions containing arccosine (x) using Maclaurin's expansion, South East Asian J. of Mathematics and Mathematical Sciences. 2020; 16(3): 83-96.

18. Qureshi MI, Malik SH, Shah TR. Hypergeometric forms of some functions involving arcsine (x) using differential equation approach, South East Asian J. of Mathematics and Mathematical Sciences. 2020; 16(2): 79-88.

19. Qureshi MI, Malik SH, Shah TR. Hypergeometric representations of some mathematical functions via Maclaurin series. 2020; 50(1): 179-188.

20. Srivastava HM, Karlsson PW. Multiple Gaussian Hypergeometric Series, Halsted Press (Ellis Horwood Limited, Chichester), John Wiley and Sons, New York, Chichester Brisbane, Toronto, 1985.

Citation: Qureshi MI, ul Rahman Shah T, Malik SH (2023) Successive differentiation of some mathematical functions using hypergeometric mechanism. Ann Math Phys 6(2): 182-186. DOI: https://dx.doi.org/10.17352/amp.000100